Absorptive Capacity and Innovation: When Is the Right Time to Cooperate?∗

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Abstract

Since Cohen and Levinthal (1989) emphasized the role of absorptive capacity in assimilating and applying new knowledge, a lot of research on the determinants of this capacity has been done. Among others, it was confirmed that proximity among cooperation partners (in terms of organizational and cognitive distance) has an inverted ‘U’-shaped relation with the value of learning the partners obtain. Furthermore, it was found that cooperation partners can increase their proximity over time and thus, affect potential of further learning. These features, however, are not jointly integrated into any theoretical model. This paper offers a revision of the original model by Cohen and Levinthal (1989) accounting for those empirical findings and constituting an appropriate framework for understanding the changes in cooperation preferences of companies. The study has important applications for both company managers and policy makers.

Keywords: Absorptive capacity; cognitive distance; cooperation intensity; innovation; proximity

1 Introduction

Absorptive capacity which allows companies to understand and apply knowledge generated elsewhere (and transferred by spillovers) is not a ‘god–given’ firm characteristic. It has to be built up continuously by investing in R&D and therefore has a key role in the firm’s innovation strategy.

Since Cohen and Levinthal (1989) emphasized the role of absorptive capacity in assimilating and applying new knowledge, a lot of research on its determinants has been

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Absorptive capacity is traditionally considered as a positive function of R&D or as a proportion of scientists and engineers in a firm. However, recent studies have shown that absorptive capacity is a multidimensional concept (Zahra and George (2002)) that cannot be appropriately proxied by R&D or staff quality alone (Flatten et al (2011)). Particularly in the context of interactive learning, absorptive capacity is also represented as a negative linear function of cognitive distance (Nootenboom et al (2007)). Here, cognitive distance describes how far apart the technology or knowledge bases of firms are. Among others, empirical studies have confirmed that proximity among cooperation partners (in terms of organizational and cognitive distance) has an inverted ‘U’–shaped relation with the value of learning the partners obtain (Wuyts et al (2005)). Furthermore, it has been argued that cooperation partners can increase their cognitive proximity over time and thus, affect the potential of further learning (Mowery et al (1996)).

Despite the existence of modeling studies that incorporate absorptive capacity (e.g., Hammerschmidt (2009); Cantner and Pyka (1998)), the foregoing empirical features are not yet integrated in a comprehensive model of inter-firm cooperation. Therefore, this paper offers a revision of the original Cohen and Levinthal (1989) model of absorptive capacity. Specifically, we adopt a dyadic cooperation framework and explicitly model absorptive capacity as a function not only of R&D, but also of the cognitive distance between cooperation partners. In doing so, we apply insights from the recent empirical studies on inter-firm cooperation, absorptive capacity and cognitive distance.

This study contributes to understanding cooperation and R&D investment preferences of companies and therefore has important applications for both company managers and policy makers. In particular, while it is widely accepted that R&D cooperation contributes to a firm’s innovative success, it remains rather unclear how to structure a portfolio of R&D agreements (Bamford and Ernst (2002); Powell (1998)) optimally. The problem of alliances and optimal R&D portfolio is most critical for companies experiencing high uncertainty on the markets (both from the demand and technology sides) (Colombo and Garrone (1998)).

Our analysis proceeds as follows. Section 2 gives an overview of existing literature, while Section 3 presents the basic model. In Section 4 the problem of investing in R&D and forming a partnership both in static and dynamic scenarios of the model are analyzed and compared to the original model by Cohen and Levinthal (1989). Section 5 presents a simulation study and finally Section 6 concludes.

2 Literature

Absorptive capacity, that is the ability to value, assimilate and apply new knowledge, was originally conceptualised as a byproduct of a firm’s R&D efforts (Cohen and Levinthal (1989)). This capacity develops a firm’s problem-solving ability (to create new knowledge
for innovation), but also allows the firm to learn or assimilate existing knowledge (for imitation) (Kim (1998)). Thus, a firm’s absorptive capacity is defined not only by its effort to create new knowledge but also by the extent to which it can access and internalise knowledge generated by others. Focusing on the latter dimension, Zahra and George (2002) extended the concept of absorptive capacity by differentiating between potential and realised absorptive capacity. Potential absorptive capacity involves the acquisition and assimilation of externally generated knowledge, while realised absorptive capacity guarantees the application of this knowledge through the development and refinement of routines that facilitate its transformation and exploitation. In this paper, the main focus is on the potential absorptive capacity. The realised capacity is considered, though, in relation to the outcomes of R&D cooperation in terms of profit.

Since the work of Cohen and Levinthal (1989), much attention has been given to the determinants of a firm’s absorptive capacity and to its possible sources. In the literature, absorptive capacity is considered as a necessity for firms to absorb knowledge spillovers. This capacity, contrary to the assumptions of some earlier models of R&D partnerships (D’Aspremont and Jacquemin (1988); Kamien et al (1992)), is considered to be a positive function of own R&D (Kaiser (2002); Kamien and Zang (2000)).

More recently, a distinction between inventive R&D and absorptive R&D has been proposed and applied to evolutionary models of technology spillovers (Cantner and Pyka (1998)) and strategic R&D investments (Hammerschmidt (2009)). Inventive R&D is the effort made by a firm to generate original knowledge, while absorptive R&D refers to the investments made to benefit from knowledge spillovers. In this context, absorptive capacity is no longer a passive byproduct of R&D, but an explicit part of firm’s strategy. This strategic necessity is even more important when the external knowledge source (from which a firm desires to learn) is not close to its prior knowledge. This is also true when the knowledge, such as that which comes from universities and research institutes, is not directly applicable to the needs of the firm. In this case, Cohen and Levinthal (1989, p. 572) argue that a firm’s capacity to appropriate the knowledge increases as the firm invests more in R&D. This argument is extended with the distinction between inventive and absorptive R&D; it can now be noted that it is not routine R&D but explicit investments in the form of absorptive R&D that facilitates the build-up of absorptive capacity. At the same time, firms need to build up a certain level of capacity to generate own knowledge through inventive R&D. Consequently, firms are faced with the strategic decision of how to optimally allocate resources between inventive and absorptive R&D, which, though complementary, are mutually exclusive. This constitutes an investment tradeoff that holds important implications for a firm’s innovation strategy.

Prior evidence has shown that inter-organisational learning is positively related to the similarity of partners’ knowledge bases (Lane and Lubatkin (1998)). In other words, firms benefit from interactive learning when their partners are cognitively closer, while cognitive
distance will constitute a disadvantage. However, this argument is limited to the context of exploitation, wherein firms are concerned with improving their performance along the same technological trajectory. In this context, a high level of mutual understanding is required to reduce transaction costs (Drejer and Vindig (2007); Cantner and Meder (2007)).

Since technological opportunities within a certain trajectory tend to decrease continuously according to Wolff’s law (Cantner and Pyka (1998)), firms seek for more explorative or extensive opportunities, the aim of which is to generate novelty. Consequently, increasing cognitive distance positively influences the value of interactive learning because it raises the novelty value of technological opportunities as well as the possibility of novel combinations of complementary resources. This is, however, only possible as long as the R&D partners are close enough to understand each other (Nootenboom et al (2007); Gilsing et al (2008)). Consequently, an understandability–novelty tradeoff emerges (Nootenboom (2000)) confronting firms with the problem of structuring an optimal portfolio of partners. The understandability–novelty tradeoff is relevant not only for cognitive distance but also for other types of distance that could exist between firms: organizational, social, institutional and geographical. According to Boschma (2005), when the distance between two partners is too little, lock-in may occur, and when it is too large coordination problems arise. Thus, effective learning by interaction is better accomplished by limiting cognitive overlap while securing cognitive proximity.

Together with the investment tradeoff described earlier, the proximity tradeoff affects both the learning potential and the innovation outcomes of cooperation. In the empirical study by Wuyts et al (2005), the value of learning that cooperation partners obtain from their partnership is an inverted ‘U’-shaped function of cognitive distance. This function is derived as the mathematical product of understandability and novelty. The main implication of this is that there exists an optimal cognitive distance at which the likelihood of generating novelty as well as finding ‘understandable’ partners is the highest. Nootenboom et al (2007) and Gilsing et al (2008) provide further results from different industries in support of the optimal cognitive distance hypothesis. In particular, they argue that the value of interactive learning and, hence, the innovation performance of a partnership, is a product of understandability and novelty. Cantner and Pyka (1998) also argue that the absorptive capacity of a firm is influenced by its distance from the frontier. Specifically, they consider an inverted ‘U’-shaped relationship.

Recent empirical contributions have shown that the cognitive overlap between partners increases with intensity of cooperation, introducing a dynamic structure on the tradeoff described. For instance, Wuyts et al (2005) argued that the cognitive distance between partners is a dynamic negative function of their frequency of interaction. In other words, the more often any pair of partners cooperate with each other, the more knowledge they share. Thus, their knowledge bases become more similar as they cooperate more
frequently. A similar argument was made by Mowery et al (1996, 1998) for the duration of cooperation. *Ex post*, technological overlap between cooperation partners will be greater than its pre–cooperation levels because each of them would mutually absorb some of the partner’s capabilities over time.

In the theoretical models of research joint ventures, the emphasis has been mostly placed on incentives to invest in R&D within cooperative settings. Earlier research in this area (e.g., D’Aspremont and Jacquemin (1988); Kamien et al (1992)) typically assume exogenous spillovers, which imply that a firm can benefit from R&D efforts of other firms without doing any R&D itself. Consequently, due to imperfect appropriability conditions, spillovers constitute a disincentive to R&D investments (Arrow (1962)). However, in later models, spillovers and absorptive capacity are introduced as endogenous parameters. Using this approach, Kamien and Zang (2000) and Kaiser (2002) find that in cooperative settings with high spillovers firms invest more than in non-cooperative settings characterised by relatively lower spillovers. In Hammerschmidt (2009), a similar approach is employed with the distinction between inventive and absorptive R&D. The results show that for high spillovers firms increase only their absorptive R&D, which allows them to maximise the benefits of external knowledge, while correspondingly reducing investments in inventive research. One can summarise these results as implying that the availability of knowledge spillovers incentivise investments in R&D, particularly to improve absorptive capacity. de Jong and Freel (2010) and Drejer and Vindig (2007) show empirically that when firms have a higher absorptive capacity (driven by R&D investments), they can reduce their cognitive distance from potential partners even if they are geographically far away.1 Consequently, firms shift themselves along the cognitive space closer to sources of higher spillovers through direct investments in absorptive capacity.

3 The Model

In the model, a firm seeks to maximize its profit from generating innovations by developing absorptive capacity to gain from knowledge spillovers while also maintaining own inventive R&D. To achieve this, the firm needs to decide how to allocate its R&D investments between own invention and the development of absorptive capacity. Knowledge spillovers are either generated voluntarily through inter-firm cooperation or involuntarily. The decision on investment allocation is affected by the firm’s distances to these two types of spillovers; larger distances correspond to higher novelty potential but also to larger investments required to absorb them. In a static scenario, the distances are given exogenously, while in a dynamic one, the distances change according to the innovation success of the firms. Each firm has to resolve the investment tradeoff as well as to select

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1 In this way, firms can overcome the constraint on the inter-firm knowledge transfer due to geographical distance (Gulati (1995); Mowery et al (1996))
an R&D cooperation partner (or, alternatively, innovate on its own).

Three important assumptions are to be noted. Firstly, in making their cooperation decisions, firms consider their potential profits only in the short term. As Cowan et al (2007) noted, this assumption is adequate when the frontier of knowledge is rapidly extending (so that firms seek to maintain competitiveness through innovation) or productive activities require a rapidly expanding knowledge base (in which case firms need to cooperate so as to gain access to complementary knowledge). Secondly, the reliability and trustworthiness of potential partners is not taken into account in the selection of cooperation partners. This follows partly from the short-termism with which firms approach partner selection. In addition, since the potential partners also have a reciprocal incentive for cooperation then their likelihood to misbehave is lower. Thirdly, firms only select one partner during a given period. This assumption improves the tractability of the model and still allows us to observe the dynamics that we are interested in.

3.1 R&D investments

Technological progress develops along certain trajectories within a given technological paradigm. Each of these trajectories contains some technological opportunities which are either intensive or extensive (Cantner and Pyka (1998)). In the former case, companies explore opportunities on a particular trajectory by investing in own R&D, whereas in the latter case, firms make use of external knowledge generated by public institutions (universities, research institutes) and other companies in the same and also other industries. For this, however, at least a share of the external knowledge must not be a private good (i.e., not appropriated by the owner), and the receiving firms must have an ability to understand and apply its content.

In the literature, there is a long discussion on the tradeoff between knowledge spillovers and appropriability conditions starting from Arrow (1962). Reducing the innovation rent, large spillover possibilities result in lower (than optimal from a social point of view) level of R&D investments. However, due to heterogeneity of companies, the knowledge transfer via these spillovers contributes to technological progress and can be beneficial for recipient firms (de Fraja (1993)). Those spillovers are nevertheless only effective if the recipient of knowledge has a sufficient absorptive capacity to make use of it.

This capacity derives from a costly investment and also depends on a firm’s prior knowledge. The relative value of the knowledge, in its turn, can be represented by the distance between the internal and external knowledge bases. If the distance is too small, companies well understand each other and there is much less uncertainty, but there might be no new knowledge to learn. In contrast, if the distance is too large, the knowledge is too difficult to absorb (Boschma (2005)).

In accordance with Cohen and Levinthal (1989), we consider R&D investments as
an instrument to stimulate absorptive capacity. However, we consider this capacity to be not a byproduct of the direct R&D investments but a separate share of the total R&D spending. This investment tradeoff is shaped by learning incentives including the potential quantity and complexity of external knowledge. Thus, we distinguish between investments directly in R&D that explore identified technological opportunities \( rdi_{i,t} \) and investments for screening surrounding technological development \( aci_{i,t} \) (Cantner and Pyka (1998)), together forming total R&D spending \( RD_i \):

\[
RD_{i,t} = rdi_{i,t} + aci_{i,t} = \rho_{i,t}RD_{i,t} + (1 - \rho_{i,t})RD_{i,t}.
\]  

\[ (1) \]

3.2 Knowledge generation

In line with Cohen and Levinthal (1989), firm \( i \)'s stock of knowledge in period \( t \) \( (k_{i,t}) \) is increased by a quantity comprising the firm’s own direct investment in R&D and externally generated knowledge which, in turn, consists of other firms’ R&D \( rdi_{k,t} \) and knowledge \( ek_{t} \) generated by public institutions:

\[
k_{i,t} = rdi_{i,t} \xi_{i,t} + aci_{i,t} \left( \delta_n \sum_{k \neq i} rdi_{k,t} + ek_{t} \right),
\]

where \( \xi \in (0, 1) \) is a parameter which defines the rate of return to inventive R&D, \( \delta_n \in (0, 1) \) reflects the fraction of knowledge not appropriated by companies\(^2\) and \( aci_{i,t} \in (0, 1) \) is the degree to which firm \( i \) can absorb external knowledge, i.e. absorptive capacity. The summation term in (2) assumes no cooperation between firms, hence no voluntary knowledge spillovers. All firms want to ensure that the value of \( \delta_n \) is as low as possible.

However, within a cooperative context the situation is different. Firms typically engage in cooperation to gain access to some of the partner’s knowledge that is otherwise inaccessible. Thus, besides only involuntary spillovers \( \delta_n \) from other firms, a firm can also appropriate voluntary spillovers \( \delta_c > \delta_n \) from its strategic partners (Gulati (1998)). But securing access to voluntary spillovers through partnerships has a potentially negative side effect for firm \( i \) because of the reciprocity that characterises cooperative arrangements. A firm \( i \)'s awareness\(^3\) of the knowledge stock of a potential partner \( j \) creates an incentive to cooperate with that partner. But in exchange, firm \( i \) also needs to open up its knowledge base (Fehr and Gächter (2000)). Consequently, spillovers from firm \( i \)'s R&D efforts reduce its own appropriation and improve its competitor’s R&D performance.\(^4\) Nevertheless, because the partner firm does not possess perfect absorptive capacity to

\(^2\)This fraction is determined by the appropriability conditions which include the patent system in a particular industry and the efficacy of secrecy or other forms of protection of firm \( j \)'s internal knowledge.

\(^3\)On the justification of this insight see (Baum et al, 2010, p. 2096).

\(^4\)This argument is important for our model and will be further discussed in conjunction with the company’s profit function.
appropriate all the spillovers, the disincentive arising from potential spillovers is lessened (Cohen and Levinthal, 1989, p. 575-6; Hammerschmidt, 2009, p.426).\(^5\) Moreover, firm \(i\) also benefits from cooperation because it has access to a pool of knowledge larger than just its own, particularly when the partner holds complementary technological knowledge thereby creating a higher potential to generate novelty.\(^6\)

Combining the foregoing arguments, it is obvious that despite the risk involved, firms can still find sufficient reasons to deliberately engage in cooperation for the purpose of gaining access to voluntary spillovers. Thus,

\[
k_{i,t} = rdi^\xi_{i,t} + ac_{i,t} \left( \delta_c + \delta_n \right) \sum_{j \neq i} rdi_{j,t} + \delta_n \sum_{k \neq i} rdi_{k,t} + ek_t \right), \quad 1 > \delta_c > \delta_n > 0.
\]

In a dyadic relationship, only one partner \(j\) is present, and it can be assumed that all involuntary spillovers available are included in the total external knowledge \(ek\). Therefore,

\[
k_{i,t} = rdi^\xi_{i,t} + ac_{i,t} \left( \delta_c rdi_{j,t} + ek_t \right).
\]

In the meantime we drop the time argument \(t\) as long as this causes no confusion.

### 3.3 Absorptive capacity

Absorptive capacity \((ac_i)\) is dependent on two variables: i) the distance \((d_i)\) between firm \(i\)'s knowledge base and external knowledge available and ii) the investments in absorptive capacity \((aci)\) made by the firm. The distance \((d_i)\) can include cognitive and organisational distances (Wuyts et al (2005)) as well as social, institutional and geographical ones (Boschma (2005)).\(^7\)

Assuming that each competing firm \(j\) maximises the portion of own knowledge that it appropriates, firm \(i\) sees the need to engage in cooperation in order to gain access to complementary resources beyond the generally available extra-industry spillovers. Thus, the choice of a cooperation partner depends on the value of the potential knowledge that can be gained and appropriated. This value refers to the distance between the knowledge base of the firm \(i\) and its potential partner \(j\) \((d_{ij})\). Initially we consider this distance to be given exogenously and fixed over time, but later we relax this assumption. The larger

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\(^5\)This mechanism reduces as the absorptive capacity of the competing firm increases.

\(^6\)In the extreme case that the cooperating partners operate in different industries, competition between them is mostly negligible; therefore, spillovers do not constitute a disincentive to cooperation and R&D investments (Cantner and Pyka, 1998, p. 374).

\(^7\)In general, we focus on cognitive distance but other types of distances could matter. For instance, Dettmann and von Proff (2010) demonstrated that organisational and institutional proximity facilitate long-distance collaboration in patenting. Similarly, Wuyts et al (2005) demonstrated that, depending on the industry, organisational and strategic proximity could be more important in the formation of alliances than cognitive proximity.
this distance, the more novel the potentially available knowledge and the higher the level of absorptive capacity investments required by firm \(i\) to appropriate it.

We proceed by considering absorptive capacity as a function of the value of knowledge absorbed from interactive learning \((an_{i,j})\) with firm \(j\). According to Wuyts et al (2005) and Nooteboom et al (2007), this has an inverted ‘U’-shaped relation to \(d_{ij}\). To the extent to which this knowledge is a function of the cognitive distance between firms, absorptive capacity emerges also as a function of cognitive distance. It is assumed that if there is no difference between firm \(i\)’s own knowledge and the external one, the novelty value is zero even if understandability is maximal. Considering absorbed knowledge as the mathematical product of novelty (increasing in the distance between knowledge bases) and understandability (that respectively decreases in \(d_{ij}\)) (Wuyts et al (2005)):

\[
an_{i,j} = (\alpha d_{ij})(\beta_1 - \beta_2 d_{ij}) = \alpha \beta_1 d_{ij} - \alpha \beta_2 d_{ij}^2
\]

and accounting for the stimulating role of investments in absorptive capacity \((aci_i)\) with \(\psi \in (0, 1)\)\(^8\) reflecting the decreasing marginal returns (Cohen and Levinthal (1989); Hammerschmidt (2009)):

\[
an_{i,j} = \alpha \beta_1 d_{ij}(1 + aci_i^{\psi}) - \alpha \beta_2 d_{ij}^2 = \alpha \beta_1 d_{ij} + \alpha \beta_1 d_{ij} aci_i^{\psi} - \alpha \beta_2 d_{ij}^2 ,
\]

absorptive capacity is presented as \(an_{i,j}\) normalized by its maximum value:

\[
aci_{i,j} = \frac{\alpha \beta_1 d_{ij} + \alpha \beta_1 d_{ij} aci_i^{\psi} - \alpha \beta_2 d_{ij}^2}{\frac{1}{4\alpha \beta_2} \left[ \alpha \beta_1 (1 + aci_i^{\psi}) \right]^2} \in [0, 1].
\]

Clearly, in (6) we refer to the absorptive capacity of firm \(i\) directed on the firm \(j\), as only the distance between these two partners is accounted: a larger \(d_{ij}\) increases the marginal impact of \(aci_i\) on absorptive capacity \((\frac{\partial aci_{i,j}}{\partial aci_i} > 0)\), which corresponds with Cohen and Levinthal (1989, p. 572).\(^9\) In contrast, the effect of \(d_{ij}\) on \(aci_{i,j}\) is ambiguous: for a given value of \(aci_i\), it is positive \((\frac{\partial aci_{i,j}}{\partial d_{ij}} > 0)\) and \(\frac{\partial^2 aci_{i,j}}{\partial d_{ij}^2} < 0\) until a certain optimal distance is reached and negative \((\frac{\partial aci_{i,j}}{\partial d_{ij}} < 0)\) otherwise. Thus, the resulting function has an inverted ‘U’-shaped relation to \(d_{ij}\) (Figure 1) and its maximum shifts right (left) with increasing (decreasing) \(aci_i\) (Figure 2), allowing a firm to adapt its absorptive capacity to the actual distance from its cooperation partner. The latter characteristic corresponds to the empirical fact that investments in absorptive capacity raise the optimal distance between cooperation partners (de Jong and Freel (2010); Drejer and Vindig (2007)).

\(^8\)\(\psi\) represents the efficiency of absorptive R&D and this investment essentially causes an upward shift in understandability for any given \(d_{ij}\).

\(^9\)Note that while \(d_{ij}\) is symmetric (i.e. identical from the point of view of both firms, \(d_{ij} = d_{ji}\)), \(an_{i,j}\) and, hence, \(aci_{i,j}\) are asymmetric, as absorptive R&D investments involved are not necessarily the same for the two companies (investment tradeoff is not solved by the two companies identically).
Figure 1: Absorptive capacity function

Figure 2: Dynamics in absorptive capacity function

Note: As company $i$ increases its investments in absorptive capacity ($ac_i$), the optimal distance to its cooperating partner increases. Thus, for the larger distance, $i$ has a higher absorptive capacity by increasing its investments (left plot). The opposite is true for the lower distance (right plot).

It should be noted that the cognitive distance of firm $i$ from firm $j$ (i.e., $d_{ij}$) and from external knowledge $ek$ (i.e., $d_{iek}$) are not necessarily the same. Thus, one would expect that the absorptive capacity directed on each of these sources of spillovers will be different.\(^\text{10}\) When this is accounted for, (3) transforms into:

$$k_i = rdi_i^\xi + ac_{i,j} (d_{c}r di_{j}) + ac_{i,ek} (ek).$$

(7)

Without an R&D partner, the knowledge to be generated by firm $i$ is different ($ek$ is the only source of external knowledge):

$$k_i^{\text{generated alone}} = rdi_i^\xi + ac_{i,ek} (ek) \quad \text{as} \quad \delta_c = 0.$$  

(8)

It is clear that when $d_i = 0$ (i.e., no novelty), absorptive capacity equals zero. This is partly because the positive effect of R&D-based absorptive capacity on growth actually

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\(^{10}\)As in (6), $ac_{i,ek} = f(d_{iek})$, although the investments in absorptive capacity from a cooperation partner and external knowledge base are the same.
arises from the interaction between endogenous R&D and technological distance. Griffith et al (2003) show some country-level empirical evidence for this argument by demonstrating that with respect to knowledge diffusion, the returns to a country’s R&D efforts only shows up when there is some technology gap between the country and the frontier. The returns also increase with increasing distance from the technology frontier. At the firm level, it can be argued that spillovers (and consequently absorptive capacity) are very low or negligible when two firms are cognitively close to each other or they are both very close to the frontier. Indeed, as Hammerschmidt (2009) has demonstrated, firms are incentivised to increase absorptive R&D only when spillovers are high; otherwise, it is more profitable for them to increase inventive R&D.

In this way, absorptive capacity \( (ac_i) \) is modeled not only to capture the ability to understand (or decode) external knowledge, but also to explicitly include the ability to explore the environment for new ideas and to identify the novel knowledge. This is why, for two identical firms in a partnership, the absorptive capacity directed on each other equals zero as there is no potential to generate novelty.

### 3.4 Innovation and profit

We assume that in each period firms possess a certain R&D budget \( RD_{i,t} \), which does not necessarily depend on sales or profits, to be distributed between inventive and absorptive R&D. Thus, we abstract from the problem of production and market competition. However, to draw a parallel with Cohen and Levinthal (1989), we introduce a profit function \( (\Pi_i) \) based on the R&D activity of the company \( i \).\(^{11}\) In particular, \( \Pi_i \) is dependent on the probability to introduce an innovation \( (\Theta) \). This probability is considered to be exogenous and reflects the uncertainty features of the innovation process (Dosi, 1988, pp. 222-223; Utterback and Abernathy (1975)).

The size of a potential innovation (if it is successful) is defined by the amount of knowledge generated in the respective period \( (k_{i,t}) \). When the firm does not form a partnership, its profit is not affected by voluntary spillovers.\(^{12}\) In a partnership, however, the profit of the firm decreases proportionally to the amount of spillovers to the partner \( j \) as well as to its absorptive capacity. Thus, comparing with Cohen and Levinthal (1989),\(^ {13}\) in our study \( \Pi_i \) is reduced not simply proportional to the knowledge generated by the partner \( (k_{j,t}) \), but to the amount of knowledge spillovers \( (ac_{j,i,t}\delta_c r di_{i,t}) \) from \( i \) that the partner \( j \) can absorb (which is essentially a constituent part of \( k_{j,t} \) that reduces the appropriability of \( k_{i,t} \)). This can be considered as a ‘cost of partnership’ affecting the choice of an R&D

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\(^{11}\)In this way, the focus of the model is narrowed to the firm’s investment decision and the generation of innovations.

\(^{12}\)Involuntary spillovers mentioned in Section 3.2 are ignored here due to i) their small size; ii) lower potential to prevent firms from appropriating results of their innovative activity.

\(^{13}\)Recall that in Cohen and Levinthal (1989) \( \frac{\partial \Pi_i}{\partial k_i} > 0 \), \( \frac{\partial \Pi_i}{\partial k_j} < 0 \) and \( \frac{\partial \Pi_i}{\partial k_i \partial k_j} < 0 \).
partner. An additional risk associated with cooperation is that of knowledge leak-out
(Müller and Herstatt (1999); Mu et al (2009); Franco and Gussoni (2010)) which often
occurs beyond what the partnership agreement stipulates and intensifies the problem of
partner selection. Accounting for this risk allows us to deal with the problem of increasing
\( \Pi_{i,t} \) for \( ac_{j,i,t} \delta_{r} r_{di,i,t} < 1 \). Therefore, for simplicity, we consider this leak-out as fixed and
equal to 1.

\[
\Pi_{i,t} = \begin{cases} 
\Theta \frac{k_{i,t}}{1 + ac_{j,i,t} \delta_{r} r_{di,i,t}} & \text{if } i \text{ has a partner } j \text{ in period } t, \\
\Theta k_{i,t} \text{generated alone} & \text{if } i \text{ has no partner in period } t.
\end{cases}
\] (9)

4 Optimal Decision Making

In the following we discuss optimal strategy of company \( i \) in i) solving the investment
tradeoff, ii) forming a partnership and iii) generating innovations in a static and dynamic
scenarios of the model. In the static case, we assume that the distance between the
cooperating firms is fixed and given exogenously. In the dynamic one, we account for the
fact that the distance between cooperating firms tends to change with respect to the fact
of cooperation between the two firms and their innovative success (\( \Pi_{i,t} + \Pi_{j,t} \)).

4.1 Investment tradeoff

For certain levels of the distance \( d_{ij} \) that maximise understandability and novelty, firm \( i \)
is incentivised to invest in absorptive R&D to maximise the amount of external knowledge absorbed. With respect to the partner’s knowledge this incentive, and consequently
the amount of investment, increases (decreases) as the cognitive distance increases (decreases).

The investment tradeoff that the firm faces is how to distribute its total R&D investment between the creation of own knowledge and the improvement of absorptive capacity optimally. This necessitates a comparison of the marginal returns to each type of investment with respect to the profit \( \Pi \) gained by the firm. For absorptive R&D to pay off, it must generate a marginal return that is equal to that of inventive R&D:

\[
\frac{\partial \Pi_{i}}{\partial ac_{i}} = \frac{\partial \Pi_{i}}{\partial r_{di}}.
\] (10)

Using (9), (7), (6) and (1), we obtain (see Appendix 1 for derivation) the condition

\[14\] With this argument one can differentiate between three types of spillovers: i) involuntary spillovers which arise due to imperfect appropriability conditions and outside cooperation; ii) voluntary spillovers which occur as a result of cooperation and within the terms of cooperative agreements; and iii) leakouts which occur as a result of cooperation but not as stipulated by the cooperative agreements.

\[15\] This relationship is easily derived from the first order conditions for equation (5).
for the R&D investment that satisfies (10)\textsuperscript{16}:

\[ F(\rho_i) = 0. \quad (11) \]

As (11) is a highly complex non-linear function with multiple local minima depending on a particular set of parameters implemented \((\alpha, \beta_1, \beta_2, \psi, \xi, RD_i, RD_j, \delta_c, ek, \rho_j \text{ and } di_{ek})\), it is a non-trivial problem to find \(\rho_i\) satisfying (11).\textsuperscript{17} For this reason we apply a heuristic optimisation technique, in particular Differential Evolution (see Appendix 2 for details).

### 4.2 Partnership formation

Since larger distances (until a certain optimum level) increase the marginal returns to new knowledge generated, it follows that each firm prefers to select a cooperation partner at the largest distance possible to maximise the novelty value of the R&D cooperation. At the same time, the partner choice is essentially constrained by understandability such that the firm \(i\) chooses a partner which it can also understand. In addition, the firm also takes into account the costs of partnership as a result of spillovers from its R&D efforts. Ultimately, the decision to cooperate (or not) is a profit-maximising one which depends on the potential profit generated when working alone in comparison with profit generated by cooperating with the most 'fitting' partner:

\[
\max \left( \Pi_{i,t}^{\text{generated alone}}, \Pi_{i,t}^{\text{with any of the possible partners}} \right). \quad (12)
\]

### 4.3 Static Scenario

Similar to Cohen and Levinthal (1989), absorbed external knowledge is endogenous and influenced by \(RD_i, d_i, \delta_c\) and \(ek\). However, in contrast to Cohen and Levinthal (1989), we make a distinction between \(aci_i\) and \(rdi_i\). Therefore, in our analysis factors that influence the firm \(i\)’s marginal returns to R&D \(\left( \partial RD_i / \partial \cdot \right)\) do not necessarily have the same effect on the marginal returns generated by the firm \(\left( \partial \Pi_i / \partial \cdot \right)\) in terms of profit. Hence, in the following we concentrate on the latter effects. In Table 1 we present our results in comparison to Cohen and Levinthal (1989), while Figure 3 illustrates them in detail for a cooperating and non-cooperating firm.

In contrast to Cohen and Levinthal (1989), where no inverted ‘U’-shaped relation to \(d_{ij}\) was assumed, we can conclude that a small distance to the partner (which does

\textsuperscript{16}For the situation where firm generates innovations without forming a partnership a different condition \((F^a(\rho_i) = 0)\) is derived, see Appendix.

\textsuperscript{17}A deterministic iterative solution (as e.g., according to the fixed-point theorem) is also not applicable as the function does not necessarily always converge to a \(\rho_i \in [0,1]\) for all possible combinations of parameters.
not require any absorptive investments) positively affects R&D profit, beyond which the company \( i \) has to invest in absorptive capacity to maintain its gain from external knowledge (the partner) reducing its investments in inventive R&D (and thus, reducing R&D profit). However, at a very large distance an ‘understandability problem’ arises, where the new knowledge cannot be absorbed as efficiently any longer (which firm \( i \) cannot compensate with sufficient investments in absorptive capacity), so that the firm may shift some share of investments back in inventive R&D. This means that companies choose a cooperation partner conditional on the investments they are ready to make in order to establish efficient collaboration.

With regard to both \( \delta_e \) and \( ek \), the situation is different. \( \partial k_i/\partial \delta_e \) and \( \partial k_i/\partial ek \) are strictly positive suggesting that the appropriability condition in a cooperative setting \( (\delta_e) \) as well as the amount of external knowledge \( (ek) \) raise the ability of the firm \( i \) to create new knowledge from external sources. Consequently, firm \( i \) is incentivised to reallocate its investments from inventive to absorptive R&D. More resources are devoted to absorptive capacity which generally results in a higher level of new knowledge \( (k_{i,t}) \) generated from the cooperation. However, in contrast to \( ek \) (which has a strictly positive effect on R&D profits), \( \delta_e \) also contributes to the spillovers the cooperating partner can potentially absorb from \( i \) reducing its R&D profit. Thus, starting from a certain level firm \( i \)’s losses from a larger \( \delta_e \) can exceed its benefits. This ambiguous inverted ‘U’-shaped relation of \( \Pi_i \) to \( \delta_e \) is necessarily affected by the absorptive R&D budget of the cooperating partner: if it small enough, firm \( i \) can benefit from intensive cooperation not being afraid that its partner absorbs much.

Finally, investment decision of the partner \( \rho_j \) has an ambiguous effect on the firm \( i \)’s investment allocation, but not on its profit (which is strictly positive). This is because as \( \rho_j \) increases, it contributes to the pool of external knowledge \( i \) can benefit from incentivising the firm to increase investments in absorptive capacity. However, for \( \rho_j \) reaching its maximum values (close to 1) the cooperating partner lowers its absorptive capacity to a very small extent. Thus, knowledge spillovers from firm \( i \) to firm \( j \) that can be absorbed do not present a big threat for firm \( i \)’s inventive R&D any longer leading to a large change in the firm’s investment allocation and, subsequently, its R&D profit.

Considering a non-cooperating firm it is clear that only \( ek \) has an effect on its investment decision and profit (as described above). What is interesting is that when we
compare it with a cooperating firm with the same set of parameters one can conclude that the non-cooperative strategy becomes lucrative when i) the distance to a potential partner is either too large (understandability problem) or too small (no novelty); ii) the cooperation intensity is either too large (threat of large spillovers) or too small (the additional external knowledge is too small to invest in it); iii) or when the partner mostly invests only in absorbing knowledge and not in its generation (free rider).

4.4 Dynamic Scenario

As much as learning and innovation are the objects of inter-firm cooperation, there exists a certain point in time when the cooperating firms might reconsider their cooperation decision. This is because the cognitive distance between partners tend to reduce with the intensity of cooperation (Wuyts et al (2005); Mowery et al (1996, 1998)) such that the
knowledge potential of any cooperation eventually becomes too low to permit recombinant novelty. Hence, the partners’ innovative capacities are negatively affected. Also at this point, investments in absorptive capacity become less productive as far as the particular cooperation is concerned. These leave the firms the choice of investing more in own knowledge generation (inventive R&D) while reducing the current level of absorptive R&D, or to search for different partners (Boschma (2005)). The tradeoffs described earlier thus arise again.

The tradeoffs arise even in an asymmetric situation where there is a leader and a follower. This is because as long as they operate within the same technological trajectory, the leading firm will have no reciprocal incentive to continue the relationship. The only incentive for the leader to continue the partnership will be that of opportunism or expropriation, which constitute disincentives for the follower.\textsuperscript{18} However, if the two companies working separately have successfully implemented new knowledge in the preceding period, their cognitive distance may increase again, so that the two firms are incentivised to re-establish their partnership:

\[
d_{ij,t} = \begin{cases} d_{ij,t-1}e^{-\eta(\Pi_i,t+\Pi_j,t)} & \text{if } i \text{ and } j \text{ are partners in period } t, \\ d_{ij,t-1}e^{\gamma(\Pi_i,t+\Pi_j,t)} & \text{otherwise} \end{cases}
\]

(13)

with \(\eta\) and \(\gamma\) being positive. In this way, the distance between the two partners changes proportional to the amount of innovations the two companies generate either together or separately, thereby affecting their own choice and the choice of the potential in the next period.

In (13), the cognitive distance between the two firms at time \(t\) is a function of their innovative activity. Starting from the distance in the previous period \(d_{ij,t-1}\), the partners may become cognitively closer to each other as long as they cooperate, but may fall further apart when they do not. The effect of new innovations being implemented \(\Pi_{t}\) is not straightforward: in the cooperative setting, they reduce cognitive distances, while in the non-cooperative one they move the firms further apart cognitively; between these two extremes there is an indifference interval during which learning occurs. Note here that the appropriability condition \((\delta_c)\), total external knowledge \((ek)\) and the cognitive distance \((d_{ij})\) are expected to show similar ambiguous effects on the profits of firms \((\Pi_i)\) in a dynamic setting. However, an important difference here is that the firm’s \(i\) expectation on the partner’s investment decision \((\rho_j)\) is not necessarily to fulfill introducing more uncertainty in the resulting dynamics. Furthermore, moving along the knowledge space according to their innovative success and the success of their partners firms may essentially form some clusters within which they exchange knowledge.

\textsuperscript{18} A problem in collaboration, especially in innovation, is that under some conditions there may be opportunities and incentives for free ridership, or for one party extracting more gain than others, or even expropriating their gain” (Nooteboom, 1999, p. 802).
5 Simulation

Since in the dynamic scenario with many firms analytical solution of our model becomes intractable, an agent based model (ABM) is introduced and simulated in this section. Agent based models have gained an increasing interest in different fields of economic research as described, e.g., in Tesfatsion (2001a) and Tesfatsion (2001b). Their main advantage is in i) a more realistic representation of agents’ behavior than in a standard representative agent model and ii) possibility of an extensive and fast simulation analysis for different parameter settings due to the ongoing advances in computational performance (Gilli and Winker (2003)).

Historically, modeling studies have treated the R&D investment and cooperation decisions of firms only with respect to exogenous spillovers (see De Bondt (1997) for a rather dated overview). Typically, such spillovers, especially when they are symmetric, have a negative effect on strategic R&D investments. At the same time, they incentivise firms to engage in cooperation and to make bilateral investment commitments. Later models account for absorptive capacity and show that technological heterogeneity, as reflected in relatively high spillover rates, incentivise the build-up of absorptive capacity in the presence of exogenous spillovers (Hammerschmidt (2009)). Even when spillovers are endogenous, as is the case in the model of Cantner and Pyka (1998), adopting a strategy that allocates more resources to absorptive R&D as spillovers increase tends to be a more profitable strategy when compared with other strategies such as one in which the firm purely concentrates on invention. A limitation of these studies is their failure to account for strategic alliance formation as a way for firms to access complementarities as well as to pool their knowledge resources or innovate jointly.

In the more recent models of Cowan et al (2007); Baum et al (2010), alliance formation is driven by its probability to succeed in terms of knowledge generation and innovation, as well as the proximity of the potential partner. Firms form bilateral partnerships which could be repeated. Over time, these partnerships result in an industry network wherein firms could have multiple partners. The patterns of partner selection and the emergent network structure as well as how these affect innovation performance are then analysed. Among others, the studies demonstrate that empirically founded network characteristics such as repeated alliances, transitivity and clustering can be observed even when alliances are formed only on the basis of knowledge considerations. They also show that innovation performance is affected by network position. However, these models treat absorptive capacity as an exogenous parameter which is similar for all firms.

Although our model shares some features of Cowan et al (2007); Baum et al (2010), an important contribution made is that absorptive capacity is not modeled exogenously, but is endogenous and is defined by two factors: a company’s distance both to a potential partner and to the external knowledge, as well as its decision on the investment tradeoff in
inventive and absorptive R&D. Thus, partner choice is driven by proximity considerations, 
enogenous absorptive capacity and the cost of partnership in terms of the knowledge 
spillovers to a potential partner that it can absorb.

5.1 Numerical experiment

In each period $t$ a company $i$ decides whether to start a new partnership and/or dissolve 
a previous one. In contrast to some literature discussed above (e.g., Baum et al (2012)), 
we consider companies forming not more than one partnership in one period $t$.

Cognitive distance $d_{ij}$ is modeled as Euclidian distance between the stock of knowl-
edge of the two partners $i$ and $j$ ($\nu_i$ and $\nu_j$), which, at the beginning, are independently 
and randomly (uniform distribution) attributed to the firms over $\kappa$ types of knowledge 
from the interval $[0, 1]$:

$$
d_{ij,t} = \sqrt{(\nu_{i1} - \nu_{j2})^2 + (\nu_{i2} - \nu_{j2})^2 + ... (\nu_{i\kappa} - \nu_{j\kappa})^2}. 
$$

(14)

We take $\kappa = 2$ for a better visualization of results, however a larger $\kappa$ might be further 
required by the model settings.$^{19}$

In our ABM (as it was done before) we assume that firms have a per fect knowledge 
about the distances to their potential partners and their R&D budgets. This assumption 
can be justified by the fact that in real world firms can assess partners’ patent portfolios, 
scientific papers (Baum et al, 2010, p.2098) and publicly available financial statements.

Now, since any particular firm takes a decision on the investments in R&D based on the 
investment decision of its potential partner, we assume that in period $t$ a firm considers 
the investment decision of the partner to be the same as in the previous period:

$$
E^i(\rho_{j,t}) = \rho_{j,t-1}. 
$$

(15)

While in the initial period $\rho$’s can be set to zero ($\forall i : E^i(rd_{i,j,2}) = RD_{j,1} \iff E^i(\rho_{j,2}) = 0$), they essentially will move from this state in the next period and adjust in a short 
period of time ($\Rightarrow$ initial periods can be then discarded from analysis).

The distance of company $i$ to the external knowledge can be measured as an average 
distance to all other firms in the knowledge space:

$$
d_{iek,t} = \frac{\sum_{i \neq k=1}^{N} d_{ik}}{N},
$$

(16)

so that the maximum distance to the external knowledge does not exceed the maximum 
distance to a single potential partner in the knowledge space. External knowledge $ek$, in

$^{19}$Similarly to Baum et al (2010, p. 2096), considering knowledge distances in a metric space in our 
ABM two companies forming a partnership in a period $t$ are presumably to continue the partnership in 
the period $t + 1$, although the probability is decreasing.
its turn, can be set in a variety of ways: i) purely exogenously and time invariant; ii) as a total inventive R&D investment of other companies in the knowledge space rescaled by the amount of involuntary spillovers (time variant):

\[ e_k = \delta_N \sum_{i \neq k=1}^{N} rd_{i,k,t}. \]  

(17)

Variable of innovation success \( \Theta \) is binary and has a lognormal probability \( \Theta \sim \ln N(\mu, \sigma^2) \) to ensure the event of innovation in any given period to be low (e.g., one innovation at any given period at maximum).

Algorithm for the ABM simulation:

1. Set all exogenous parameters: \( \alpha, \beta_1, \beta_2, \psi, \xi, \eta, \gamma, \delta_c, e_k \) (the latter two can be simulated with different scenarios).

2. Distribute randomly the initial stocks of knowledge for all firms \( \Rightarrow \) set \( d_{ij} \) for \( t = 1 \) and set aggregated R&D budgets \( RD \) for all firms.\(^{20}\)

3. In each round do the following:
   - solve an investment tradeoff of company \( i \) \( (\rho_i) \) for all potential partners considering their investments in R&D based on the expectations from the past period;
   - estimate the amount of knowledge \( k_{i,t} \) to be generated by company \( i \) at the period \( t \) either alone (standalone mode) or in partnership with any of the company;
   - identify the most lucrative partner for each by maximizing profit\(^{21}\) from R&D activity \( \Pi_i \): \( \max(\Pi_{i,\text{alone}}, \Pi_{i,\text{..}}) = \max(\Theta k_{i,t}, \Theta 1 + ac_{i,i,t} \delta_c rd_{i,t}, ...) \);
   - form partnerships (alternative algorithms are described below);
   - estimate companies’ profits based on realized innovation success \( (\Theta) \);
   - recalculate the distances based on the choice of a partner \( d_{ij,t+1} \forall i \text{ and } j \).

4. After a pre-specified number of periods \( T \) stop the code and display results on:
   - location of companies (over time);
   - their generated profits;

\(^{20}\)Alternatively, one can randomly attribute \( RD \) for all firms at each period of time from a certain interval.

\(^{21}\)The initial idea is to consider the accumulation of profits by firms over the number of periods \( \sum_{t=1}^{T} \Pi_t \) dependant on the other factors like the amount of external spillovers \( e_k \), intensity of cooperation \( \delta_c \),... However, potentially one can invest these profits into generation of knowledge in the next period, i.e. add them to \( RD_i \).
frequency and history (structure) of cooperation.

The incentives of a firm \( i \) to build a partnership with firm \( j \) are asymmetric since, although distance between the partners is the same, the decision on the investment tradeoff in R&D is individual for each company. Hence, there is no ‘Nash stable network’ (‘a stable network is one in which for each agent (or pair of agents) there is a payoff maximizing decision about which link to form’ (Cowan et al, 2007, p. 1052)). So far, we consider few alternatives on forming partnerships:

- A rule of thumb: if firm \( i \) identifies firm \( j \) as the most lucrative cooperation partner and is itself among ‘top’ 10% of the companies with whom firm \( j \) would cooperate, then they build a partnership. The main advantage of the method is its simplicity and low computational time required;

- An algorithmic search: first, we exclude all firms out of the further search, which in any case prefer ‘standing alone’ (no cooperation); then we randomly select a company \( i \) out of the population and identify its most lucrative partner \( j \). If for \( j \) it is the same (\( i \) is most lucrative), they form a partnership and are excluded from further search. If it is not the case, we continue with \( j \) searching for its most lucrative partner. If the search path becomes cyclical (\( i \rightarrow j \rightarrow k \rightarrow i... \)), we exclude the entire cycle (none of the firms builds a partnership) and continue the random search. Computational time is high and increases exponentially with the number of firms in the population.

5.2 Results

tba

6 Conclusions and outlook

In this paper we set out to model absorptive capacity within the framework of inter-firm cooperation such that the capacity of a firm to appropriate external knowledge is not only a function of its R&D efforts but also of the distance from its partner. This framework allows us to account for recent empirical observations and to examine the dynamics of inter-firm cooperation. Our analysis is restricted to cognitive distance though other kinds of distance are also important. We analyse two situations: the static case where the cognitive distance between cooperating firms is fixed and given exogenously, and the dynamic case where distance varies as a function of the intensity of cooperation between partners.

Comparing the results so far with the original models of Cohen and Levinthal (1989) show some clear differences. In the static case, the cognitive distance between a firm and
its cooperation partner has an ambiguous effect on the profit generated by the firm. A firm will choose a cooperation partner conditional on the investments it is willing to make to maximise understandability and the innovative potential of cooperation. Similarly, appropriability conditions in the framework of cooperation also has an ambiguous effect on profits. Starting from a certain threshold, the more spillovers a firm generates, the more its partner could potentially benefit and erode its profit. In contrast, total external knowledge, that is external knowledge available outside the framework of cooperation, as well as the partner’s inventive R&D investments have positive effects on profit.

In the dynamic case, it is expected that a firm will reconsider its cooperation decisions depending on cognitive distance. Alliances may be discontinued when partners become too close and previously discontinued alliances may be re-formed if the partners have become sufficiently distant in terms of their knowledge endowment. A simulation experiment to demonstrate this dynamics is in progress.

**Appendix 1**

**Derivation of the condition for** $F(\rho_i) = 0$ (equation (11))

The objective is to obtain values of $\rho_i$ that satisfy:

$$\frac{\partial \Pi_i}{\partial a_{ci}} = \frac{\partial \Pi_i}{\partial r_{di}}.$$  

Recall from (9) that in case of a partnership, where $i$ needs to optimise its investment allocation conditional upon the partner’s investments, $\Theta \frac{k_i}{1 + a_{cji} \delta_c r_{di}}$. The time argument is dropped since for now we consider the static scenario:

$$\frac{\partial \Pi_i}{\partial r_{di}} = \frac{\partial (\frac{k_i}{1 + a_{cji} \delta_c r_{di}})}{\partial r_{di}} = \frac{\xi r_{di}^{\xi-1}(1 + a_{cji} \delta_c r_{di}) - k_i \delta_i a_{cji}}{(1 + a_{cji} \delta_c r_{di})^2},$$  \hspace{1cm} (18)

$$\frac{\partial \Pi_i}{\partial a_{ci}} = \frac{\partial \left(\frac{k_i}{1 + a_{cji} \delta_c r_{di}}\right)}{\partial a_{ci}} = \frac{(1 + a_{cji} \delta_c r_{di}) \left(\frac{\partial k_i}{\partial a_{ci}}\right) + k_i \delta_i a_{cji}}{(1 + a_{cji} \delta_c r_{di})^2},$$  \hspace{1cm} (19)

where $E^i(\rho_{j,t}) = \rho_{j,t-1} \Rightarrow \frac{\partial a_{cji}}{\partial r_{di}} = 0$ and $r_{di} = RD_i - a_{ci} \Rightarrow \frac{\partial r_{di}}{\partial a_{ci}} = -1$. Next we set (18) equal to (19) as in equation (10):

$$\xi r_{di}^{\xi-1}(1 + a_{cji} \delta_c r_{di}) - k_i \delta_i a_{cji} = (1 + a_{cji} \delta_c r_{di}) \left(\frac{\partial k_i}{\partial a_{ci}}\right) + k_i \delta_i a_{cji}$$
and collect terms:

\[
\frac{\xi rdi_i^{\xi-1}}{\left(\frac{\partial k_i}{\partial ac_i}\right)} = \frac{2k_i \delta_i ac_j,i}{(1 + ac_j,i \delta rdi_i)}. \tag{20}
\]

Recalling the expression for \(k_i\) from (7) we obtain

\[
\frac{\partial k_i}{\partial ac_i} = \delta_i rdi_j \left(\frac{\partial ac_{i,j}}{\partial ac_i}\right) + ek \left(\frac{\partial ac_{i,ek}}{\partial ac_i}\right). \tag{21}
\]

Accounting for the difference in \(d_{ij}\) and \(d_{iek}\) in \(ac_{i}\), (6) we obtain the derivative of the absorptive capacity function with respect to distance as follows:

\[
\frac{\partial ac_{i}}{\partial ac_i} = \frac{4 \beta_2 \psi d_i ac_i^{\psi-1} - 1}{\beta_1(1 + ac_i^{\psi})^2} \] 

\[
\tag{22}
\]

Inserting (22) into (21) accordingly:

\[
\frac{\partial k_i}{\partial ac_i} = \delta_i rdi_j \left(\frac{4 \beta_2 \psi d_i ac_i^{\psi-1} - 1}{\beta_1(1 + ac_i^{\psi})^2} \right) + ek \left(\frac{4 \beta_2 \psi d_{iek} ac_i^{\psi-1} - 1}{\beta_1(1 + ac_i^{\psi})^2} \right). \tag{23}
\]

Note that the absorptive capacity of firm \(j\) directed on firm \(i\) is:

\[
ac_{j,i} = \frac{\alpha \beta_1 d_{ij} + \alpha \beta_1 d_{ij} ac_i^{\psi} - \alpha \beta_2 d_{ij}^2}{\frac{1}{4\alpha \beta_2} \left[\alpha \beta_1(1 + ac_i^{\psi})\right]^2} \] as \(d_{ij} = d_{ji}. \tag{24}
\]

When (23) and (24) are inserted in (20) and the latter is rearranged, we obtain

\[
rdi_i = \frac{32 \beta_2^2}{\xi \alpha \beta_1^4 \left(\beta_1 + \beta_1 ac_i^{\psi} - \beta_2 d_{ij}\right)^2 \left(1 + ac_i^{\psi}\right)^5} \delta_i rdi_j d_{ij} \left(2 \beta_2 d_{ij} - \beta_1 \left(1 + ac_i^{\psi}\right)\right) +
\]

\[
+ ek d_{iek} \left(2 \beta_2 d_{iek} - \beta_1 \left(1 + ac_i^{\psi}\right)\right) \frac{ac_i^{\psi-1}}{rdi_i^{\xi-1}} \left(\beta_1 + \beta_1 ac_i^{\psi} - \beta_2 d_{ij}\right).
\]

\[
\cdot \left(\frac{rdi_i^\xi}{4\alpha \beta_2} \left(\alpha \beta_1 \left(1 + ac_i^{\psi}\right)\right)^2 + \alpha \delta_i rdi_j d_{ij} \left(\beta_1 + \beta_1 ac_i^{\psi} - \beta_2 d_{ij}\right) +
\]

\[
+ \alpha d_{iek} ek \left(\beta_1 + \beta_1 ac_i^{\psi} - \beta_2 d_{iek}\right) \right) \frac{\beta_1 \left(1 + ac_i^{\psi}\right)^2}{4\beta_2 \delta_i d_{ij} \left(\beta_1 + \beta_1 ac_i^{\psi} - \beta_2 d_{ij}\right)}. \tag{25}
\]
Recall from (1) that $rd_i = \rho_i RD_i$ and $ac_i = (1 - \rho_i) RD_i$; when this is applied to equation (25) it takes the form:

$$
\rho_i = \frac{32 \beta^2}{\xi \alpha \beta_1^4 RD_i \left( \beta_1 + \beta_1 \left( (1 - \rho_j) RD_j \right)^\psi - \beta_2 d_{ij} \right) \left( 1 + (1 - \rho_i) RD_i \right) \psi} 
\cdot \left( \delta c \rho_j RD_j d_{ij} \left( 2 \beta_2 d_{ij} - \beta_1 \left( 1 + (1 - \rho_i) RD_i \right)^\psi \right) \right) + 
+ ek d_{iek} \left( 2 \beta_2 d_{iek} - \beta_1 \left( 1 + (1 - \rho_i) RD_i \right)^\psi \right) \frac{(1 - \rho_i)^{\psi - 1} RD_i^{\psi - \xi}}{\rho_i^{\xi - 1}}. 
\cdot \left( \beta_1 + \beta_1 \left( (1 - \rho_j) RD_j \right)^\psi - \beta_2 d_{ij} \right) \left( \frac{(\rho_i RD_i)^\xi}{4 \alpha \beta_2} \left( \alpha \beta_1 \left( 1 + (1 - \rho_i) RD_i \right)^\psi \right) \right)^2 + 
+ \alpha \delta c \rho_j RD_j d_{ij} \left( \beta_1 + \beta_1 \left( (1 - \rho_i) RD_i \right)^\psi - \beta_2 d_{ij} \right) + 
+ \alpha d_{iek} ek \left( \beta_1 + \beta_1 \left( (1 - \rho_j) RD_j \right)^\psi - \beta_2 d_{iek} \right) - 
- \frac{\beta_1 \left( 1 + (1 - \rho_j) RD_j \right)^\psi}{4 \beta_2 \delta c d_{ij} RD_i \left( \beta_1 + \beta_1 \left( (1 - \rho_j) RD_j \right)^\psi - \beta_2 d_{ij} \right)}. 
(26)
$$

Shifting $\rho_i$ from the left hand side to the right one, one gets $F(\rho_i) = 0$.

Remembering that for firm $i$ performing R&D activity without a partner $\delta_c = 0$, it is straightforward to show that for this firm (26) takes a simpler form as follows:

$$
F^a(\rho_i) = ek \frac{4 \beta_2 \psi d_{iek} (1 - \rho_i) RD_i)^{\psi - 1}}{\beta_1 \left( 1 + (1 - \rho_i) RD_i \right)^{\psi \nu}} \left( \frac{2 \beta_2 d_{iek}}{\beta_1 \left( 1 + (1 - \rho_i) RD_i \right)^{\psi \nu} - 1} \right) - \xi (\rho_i RD_i)^{\xi - 1} = 0.
(27)
$$

### Appendix 2

#### Differential Evolution

Thanks to the recent advances in computing technology, new nature-inspired optimization methods (called heuristics) tackling complex combinatorial optimization problems and detecting global optima of various objective functions have become available (Gilli and Winker (2009)). Differential Evolution (DE), proposed by Storn and Price (1997), is a population based optimization technique for continuous objective functions. In short, starting with an initial population of solutions, DE updates this population by linear combination and crossover of four different solutions into one, and selects the fittest ones among the original and the updated population. This continues until some stopping
criterion is met. Algorithm 1 provides a pseudocode of the DE implementation.

**Algorithm 1 Pseudocode for Differential Evolution**

1: Initialize parameters $p$, $F$ and $\Omega$
2: Randomly initialize $P_i^{(1)} \in \Omega$, $i = 1, \ldots, p$
3: while the stopping criterion is not met do
4:   $P^{(0)} = P^{(1)}$
5:   for $i = 1$ to $p$ do
6:     Generate $r_1, r_2, r_3 \in [1, \ldots, p]$, $r_1 \neq r_2 \neq r_3 \neq i$
7:     Compute $P_i^{(\nu)} = P_i^{(0)} + F \times (P_{r_1}^{(0)} - P_{r_2}^{(0)})$
8:     if $P_i^{(\nu)} \in \Omega$ then $P_i^{(n)} = P_i^{(\nu)}$ else repair $P_i^{(\nu)}$
9:     if $F(P_i^{(n)}) < F(P_i^{(0)})$ then $P_i^{(1)} = P_i^{(n)}$ else $P_i^{(1)} = P_i^{(0)}$
10:   end for
11: end while

In contrast to other DE applications to optimization problems (as, e.g., described in Blüschke et al (2012)), our solution is represented by a single value within $[0, 1]$ according to (1). Therefore, DE starts with a population of size $p$ of random values drawn from $[0, 1]$ ($\Omega$) (2). For the same reason, current DE implementation has no need in the crossover operator (otherwise, one would have to compare $F(P_i^{(0)})$ with itself and potentially waste computational time). Tuning our DE code we set $p = 30$, $F = 0.8$ and as a stopping criterion we choose a combination of two conditions: either a maximum number of generations is reached (which is set to be equal 50\textsuperscript{22}) or the global optimum is identified ($F(P_i^{(1)}) = 0$). To make sure that our candidate solutions constructed by linear combination (7:) satisfy our constraint on $\rho_i$, we explicitly check it in (8:) - and if it is not met we ‘repair’ it by adding/deducting one unit - before comparing its fitness with the current solutions in (9:).

As an illustration of the DE convergence for the tuning parameters stated consider Figure 4 below.\textsuperscript{23} On the left plot one can see $F(\rho_i)$ simulated for different $\rho_i \in [0, 1]$, while on the right plot cumulative density function of $F(\rho_i)$ for 100 restarts and different number of maximum generations $g$ (10, 30 and 50) is given. As it clear, with $g = 50$ DE converges to zero (or a very close approximation of it) in almost 100% of restarts. To ensure a good solution, therefore, we take $g = 30$ and restart DE three times. Using Matlab 7.11 on Pentium IV 3.3 GHz a single DE restart with thirty generations requires about 0.02 second.

\textsuperscript{22}At this point DE population always converges to very similar values

\textsuperscript{23}For illustrative reasons we take a single set of parameter values satisfying their constraints. In particular, $\alpha = \sqrt{2}/50$, $\beta_1 = \sqrt{2}$, $\beta_2 = 1$, $\psi = 0.3$, $\xi = 0.5$, $RD_i = RD_j = 4$, $\delta_c = 0.75$, $ek = 1$, $\rho_j = 0.5$ and $d_{ik} = \sqrt{2}/1.01$.  

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Figure 4: $F(\rho_i)$ simulated for different $\rho_i$ and empirical distribution of $F(\rho_i)$ for different $g$

References


