Optimal Long-Term Allocation
for a Defined-Contributions Pension Fund

Eric Jondeau\textsuperscript{a} and Michael Rockinger\textsuperscript{b}

January 2014

Abstract

We build a macroeconomic model for Switzerland, the Euro Area, and the USA, which drives the dynamics of several asset classes and the liabilities of a representative Swiss (defined-contributions) pension fund. This encompassing approach allows us to generate correlations between returns on assets and liabilities. We calibrate the economy using quarterly data between 1985:Q1 and 2013:Q2. Using a certainty equivalent approach, we show that a liabilities-hedging portfolio outperforms an assets-only strategy by between 5 and 15% per year. The main reason for such a large improvement is that the optimal assets-only portfolio is typically long in cash, whereas hedging liabilities require the pension fund to be short in cash. It follows that imposing positivity restrictions in the construction of the portfolio also results in a large cost, between 4 and 8% per year. This estimate suggests that allowing pension funds to hedge their liabilities through borrowing cash and investing in a diversified bond portfolio would help enhancing global portfolio return.

Keywords: Asset Liability Management, Defined Contributions, Switzerland, Discounting, Markov Model, Surplus maximization.

JEL Classification: E43, C53, G23, G28

\textsuperscript{a}Swiss Finance Institute and University of Lausanne, Faculty of Business and Economics, CH 1015 Lausanne, Switzerland. E-mail: eric.jondeau@unil.ch. Corresponding author.

\textsuperscript{b}Swiss Finance Institute and University of Lausanne, Faculty of Business and Economics, CH 1015 Lausanne, Switzerland. E-mail: michael.rockinger@unil.ch.

The typical disclaimer applies.
1 Introduction

Over the last five years, most developed countries have experienced a low return environment with particularly low short-term interest rates. For instance, the Swiss three-month interbank rate is below 0.5% since the first quarter of 2009. This new environment raises new issues for pension fund management. Most pension funds have been rather generous with their retirees. Contributing employees have received large promises. With increasing life expectancy and low returns, it is now more difficult to keep those engagements and promises. For these reasons, several pension funds have already changed their policy (from defined benefits plans to defined contributions plans, for instance), and the technical rate and conversion rate, which define the commitment of pension funds toward insured beneficiaries, have been regularly decreased over the recent period.

Although the difficulty to generate return is not specific to pension funds, this industry is particularly prone to it. Interest-rate risk being the main source of risk for pension liabilities, standard hedging techniques suggest to hold a large amount of long-term debt, generating low returns. Since stocks also struggle to yield high returns, the funded ratio of most pension funds has worsened since the crisis.

As standard cash-flow matching and immunization perform poorly in a low return environment, pension funds have recently considered alternative portfolio allocation approaches, such as surplus optimization and liabilities-driven investment (LDI), to reconcile liabilities hedging and performance enhancing. The main objective of this paper is to evaluate the ability of mixed approaches to generate sufficient return in the current low interest-rate environment.

In order to evaluate these approaches, several issues need to be addressed. Firstly, we need to describe the dynamics of the return on liabilities. In a low return environment,
some factors that are usually considered negligible may play a more important role. Typically, the dynamics of the pension-fund population or trends in price and wage inflation may require different investment strategies. Also pension fund with a growing or declining population may have different optimal investment policies. Therefore, the approach sometimes found in the literature, consisting of proxying return on liabilities with long-term government bond rate should be improved. Thus, we need to describe the dynamics of liabilities using a model for the pension fund. Secondly, academic research on surplus maximization and LDI generally assumes no restriction on portfolio weights. This is particularly important because these approaches are designed to form a hedge portfolio with a priori long and short investments. In practice, pension funds have many regulatory restrictions and may not be able to invest in portfolios with short investment. It is therefore important to analyze these strategies taking portfolio weight restrictions into account. Last, it is common practice to discount pension fund’s liabilities using either past interest rates smoothed over some period of time (actuarial valuation) or term-structure forecasts (economic valuation). In the current context, where interest rates are historically low, this may have important consequences for the optimal portfolio allocation because smoothed discount rates will first over-estimate and then under-estimate actual interest rates. As we are interested in the paper in the optimal financial behavior of the pension fund, we consider its actual economic situation. As a consequence, we adopt a fully economic point of view, in which the discounting is based on term-structure forecasts. We also view the plan population as an open group, so that its dynamics and renewal have to be described.

In the empirical part of the paper, we investigate the optimal long-term investment process of a representative Swiss pension fund.\textsuperscript{1} Most Swiss pension funds have defined-contributions plans, in which benefits are defined as a fraction of the accumulated saving and therefore depend on the ability of pension-fund managers to generate a sufficient return on assets. In principle, in a defined-contributions plan, the risk is not borne by

\textsuperscript{1}The specific aspects of the Swiss system have been described in Büttler and Ruesch (2007) or Von Ah (2010).
the sponsor but by the insured people. Thus for the pension fund, managing assets in relation to liabilities may not be a priority. In practice, during the financial crisis, the low performance of pension funds in terms of asset management has worsened their global situation: as of 2011, 30% of Swiss pension funds have a surplus ratio below 100%. Unfunded liabilities amount to 7.3 billion CHF for private funds and 35 billion for public funds (OFS, 2013). Designing an investment strategy that is able to – at least – match liabilities is nowadays an important objective.

We estimate an international macro-finance model, which allows us to forecast returns and risks of Swiss and international asset classes. The model is a stationary restricted Vector Error Correcting model generating predictions of macroeconomic and financial factors. As returns on assets and liabilities are related through the factors generated by the macro-finance model, liabilities risk can be partly hedged by investing in an appropriate combination of assets. This allows us to investigate various important questions in a controlled environment: (1) What is the cost of allocating assets without considering their relation to liability, i.e., ignoring liability hedging. (2) What is the cost of restrictions on portfolio weights? (3) How does the evolution of the pension-fund population affect the optimal allocation?

We evaluate the cost of suboptimal allocation strategies using the certainty equivalent. We provide evidence that neglecting liabilities-hedging portfolio results in a cost of between 5 and 15% per year, depending on risk aversion. The main reason for such a large cost is that the optimal assets-only portfolio is typically long in cash, whereas hedging liabilities require the pension fund to be short in cash. It follows that imposing positivity restrictions in the construction of the portfolio also results in a large cost, between 4 and 8% per year. These estimates suggest that allowing pension funds to hedge their liabilities through borrowing cash and investing in a diversified bond portfolio would help enhancing global portfolio return.

---

2We consider only positivity constraints. Actual pension funds may be subjected to more stringent conditions limiting the amounts invested abroad or the amounts allocated to some asset classes.
The rest of the paper is organized as follows. In Section 2, we describe the different blocks of our model: the macro-finance model, the modeling of the assets and liabilities, and the optimal allocation strategy. In Section 3, we present the data and a preliminary investigation of the liabilities and the hedging properties of the assets. In Section 4, we discuss results regarding the optimal allocation of the pension fund under different scenarios. Section 5 concludes.

2 Model

In Figure 1, we display the various blocks of our global model. First, a macro-finance model describes the dynamics of the important macroeconomic and financial factors (output growth, inflation, wages, term structure, etc.). These factors serve as input to the financial assets forecasts model to generate the expected returns, volatilities, and correlations of the various asset classes over long-term horizons.

As shown on the right-hand-side of the figure, the pension-fund model combines the macroeconomic and financial factors with demographics and regulatory parameters (contribution rate, conversion rate, etc.) to generate forecasts of population and cash flows. By discounting future cash flows with term-structure forecasts, our model generates forecasts of liabilities and eventually of returns on liabilities.

As the figure also illustrates, both financial forecasts and returns on liabilities are brought together in the allocation module to construct assets-only and assets-liabilities optimal portfolios. Dependence between returns on assets and returns on liabilities, which determines the hedging properties of financial assets, is estimated by combining the macro-finance model and the pension-fund model through Monte-Carlo simulations. In this module, we investigate the economic consequences of imposing or relaxing portfolio weight restrictions. Once we obtain the optimal portfolio allocation, we evaluate the resulting pension fund performance in the last module. The rest of this section describes the main aspects of the various components.
2.1 Macro-Finance Model

Our macro-finance model considers the U.S.A., the Euro Area, and Switzerland, as Swiss pension funds are likely to concentrate their allocation in these regions.\(^3\) We need a closed model in order to perform long-term forecasting and simulation. The main variables described by the model are: real GDP, GDP deflator, consumption price inflation, wage and employment growth, unemployment rate, short- and long-term rates, price-dividend ratio, currencies, and commodity prices.

The model is based on the error-correction principle (Engle and Granger, 1987). As all variables in level are non-stationary, their interactions are modeled through cointegration relations. Given the number of variables involved in the model (30), we cannot estimate an unrestricted VAR(1) because the resulting number of unknown parameters would be too large (30 \(\times\) 30 for the lag parameter matrix and 30 \(\times\) 31/2 for the covariance matrix). Instead we estimate a restricted version, in which explanatory variables are selected based on economic theory and on statistical significance. In the resulting model, all parameter estimates have the sign and magnitude predicted by theory and are highly significant. Residuals of the cointegration relations are stationary and represent driving forces toward the equilibrium levels described in the cointegration relations. The short-run dynamics of the model for the variables in difference is also driven by macroeconomic and finance theory.\(^4\) Stationarity of the model is guaranteed by the use of an error-correction model, in which all the residuals of the cointegration relations are stationary. The error-correction model is written as:

\[
A_0 \Delta X_{t+1} = \hat{\mu} + A_1 \Delta X_t + A_2 X_t + A_3 \varepsilon_{t+1}, \tag{1}
\]

\(^3\)Our setting can be adapted to other countries.

\(^4\)A natural alternative representation of this macro-finance model would be a dynamic stochastic equilibrium (DSGE) model. Calès, Jondeau, and Rockinger (2013) have investigated such a model for long-term asset allocation. Although explicitly modeling economic and financial theoretical relations in a complete model is very appealing, it also raises several empirical issues that render the international extension out of reach. In this paper, we rely on a simplified, reduced-form, version of this model.
where $\Delta X_t$ contains the $n$ stationary variables of the system and the innovations vector, $\varepsilon_{t+1}$, has zero mean and identity covariance matrix. Matrices $A_0$, $A_1$, and $A_2$ have restrictions imposed by economic theory and matrix $A_3$ captures contemporaneous correlation among the variables. Closing the model in a VAR form yields:

$$
\begin{bmatrix}
A_0 & 0_n \\
-I_n & I_n
\end{bmatrix}
\begin{bmatrix}
\Delta X_{t+1} \\
X_t
\end{bmatrix}
= 
\begin{bmatrix}
\tilde{\mu} \\
0_n
\end{bmatrix}
+ 
\begin{bmatrix}
A_1 & A_2 \\
0_n & I_n
\end{bmatrix}
\begin{bmatrix}
\Delta X_t \\
X_t
\end{bmatrix}
+ 
\begin{bmatrix}
A_3 & 0_n \\
0_n & 0_n
\end{bmatrix}
\varepsilon_{t+1},
$$

or with obvious notations:

$$B_0 Y_{t+1} = \mu + B_1 Y_t + B_2 \eta_{t+1}. \quad (3)$$

Solving the model shows that the final model is a stationary, parsimonious, restricted VAR(1) model:

$$Y_{t+1} = \Phi_0 + \Phi_1 Y_t + \Phi_2 \eta_{t+1}, \quad (4)$$

where $\Phi_0 = B_0^{-1} \mu$, $\Phi_1 = B_0^{-1} B_1$, and $\Phi_2 = B_0^{-1} B_2$. Matrix $\Phi_2$ captures contemporaneous correlation among the variables, and $\Sigma = \Phi_2 \Phi_2'$ is the covariance matrix of the error term.

With this model, we are able to forecast macroeconomic and financial factors, which in turn drive the dynamics of returns on assets and liabilities. A general description of the relations in the macro-finance model is given in Figure 2. It shows, for a given country, how macroeconomic and financial factors interact to predict financial assets and liability dynamics. Arrows indicate causal links from one variable to the others. For instance, output gap helps predicting inflation, employment, short-term rate, and the dividend-price ratio.

This model allows us to compute the correlations between returns on assets and liabilities, which are then used for the assets-liabilities allocation. It is also used for estimating the uncertainty surrounding our estimates. Using Monte-Carlo simulations of the macro-finance model, we can infer confidence intervals for expected returns and
portfolio weights. We can also deduce risk measures, such as Value-at-Risk and expected shortfall of the pension-fund surplus.

2.2 Modeling Assets

The most common assets are cash, bonds, and equities within each country. We relate the dynamics of asset returns to the macro-finance model. The monetary policy reaction function typically describes the response of the short-term interest rates \( r^{(3m)}_{t+1} \) to output gap \( og_{t+1} \) and inflation \( \pi_{t+1} \) pressures (Taylor, 1993). The equilibrium relation is written as:

\[
r^{(3m)}_{t+1} = \mu_{r,0} + \mu_{r,1} \pi_{t+1} + \mu_{r,2} og_{t+1} + \varepsilon_{r,t+1}. \tag{5}
\]

The short-run dynamics describes the evolution of the short-term rate toward this long-run equilibrium level:

\[
\Delta r^{(3m)}_{t+1} = \mu_{\Delta r,0} + \mu_{\Delta r,1} \varepsilon_{r,t} + \mu_{\Delta r,2} \Delta \pi_{t+1} + \mu_{\Delta r,3} \Delta og_{t+1} + \varepsilon_{\Delta r,t+1}. \tag{6}
\]

As suggested by the expectations hypothesis of the term structure and the Fisher relation, the 10-year government bond rate \( y^{(10)}_{t+1} \) is driven by the short-term rate, with a term premium that varies with inflation and growth expectations:

\[
y^{(10)}_{t+1} = \mu_{y,0} + \mu_{y,1} r^{(3m)}_{t+1} + \mu_{y,2} \pi_{t+1} + \mu_{y,3} og_{t+1} + \varepsilon_{y,t+1}. \tag{7}
\]

Holding period return is obtained as:

\[
r^{(10)}_{t+1} = D^{(10)}_t y^{(10)}_t - D^{(9)}_{t+1} y^{(9)}_{t+1}, \tag{8}
\]

where \( D_t^{(k)} = (1 - \exp(-ky_t^{(k)}))/(1 - \exp(-y_t^{(k)})) \) is Macaulay’s duration of a bond of maturity \( k \) years.
Real stock return dynamics depends on the evolution of the log dividend price ratio (dpr) (Campbell and Shiller, 1988 and 2001, Barberis, 2000). The dpr is in fact non-stationary and needs to be described by a cointegration relation. The main determinant of dpr is found to be the real long-term interest rate:

$$dpr_{t+1} = \mu_{dpr,0} + \mu_{dpr,1} (y_{t+1}^{10} - \pi_{t+1}) + \varepsilon_{dpr,t+1}, \tag{9}$$

and the dynamics of real stock return is given by:

$$\rho_{s,t+1} = \mu_{s,0} + \mu_{s,1} \varepsilon_{dpr,t} + \mu_{s,2} \Delta(r_{t+1}^{(3m)} - \pi_{t+1}) + \mu_{s,3} \Delta(r_{t+1}^{(10)} - \pi_{t+1}) + \cdots + \varepsilon_{s,t+1}. \tag{10}$$

These various relations provide a rather good description of U.S. asset returns. For the Euro Area and Switzerland, we need to account for international arbitrage. It is clear that long-term rates and stock returns in Europe are also driven to some extent by U.S. factors. For this reason, we also introduce U.S. variables as potential drivers of European financial asset returns. By proceeding in this way, we capture the well-known contemporaneous correlation between these variables, while retaining an economic interpretation of the estimated relations.

Finally, currency hedging may be an important issue for pension funds, in particular in a context where large currency fluctuations may offset asset returns. For instance, over the recent period, the Swiss Franc has experienced large fluctuations against the Euro and U.S. Dollar. For this reason, we assume that all investments in U.S. and European assets are fully hedged. Even with full currency hedging, exchange rates have to be modeled because they may be important drivers of monetary policy and/or international arbitrage. For instance, the evolution of the Swiss short-term rate has been partly driven by the Euro-Swiss Franc exchange rate over the last years.

In addition to the classical assets described above, we also added commodities and real estate as investment vehicles for pension funds. These asset classes are known to be somewhat uncorrelated to classical assets and may therefore provide good diversification.
opportunities to pension funds in terms of diversification. Modeling commodities and real estate returns is a difficult task in a simplified macroeconomic framework. Both asset returns are found to be related to economic growth: global (U.S. and European) growth for commodities, regional (European and Swiss) growth for Swiss real estate.

We denote the nominal log-return of asset \( i \) between \( t \) and \( t+1 \) by \( r_{i,t+1} \) and the real log-return by \( \rho_{i,t+1} = r_{i,t+1} - \pi_{t+1} \). Even if the three-month T-bills rate is risky in the long run, we maintain the usual distinction between the three-month rate, \( r_{t+1}^{(3m)} \), and the other returns on risky assets, grouped in \( r_{t+1} = (r_{1,t+1}, \ldots, r_{N,t+1})' \). We also define the excess log-returns relative to the three-month rate as \( x_{i,t+1} = r_{i,t+1} - r_{t+1}^{(3m)} \), \( i = 1, \ldots, N \). The vector \( x_{t+1} \) is linear in macroeconomic and financial factors \( (Y_{t+1}) \) through the relation: \( x_{t+1} = M Y_{t+1} \), where \( M \) is a selection matrix.

The expected excess log-returns and associated covariance matrix at \( t+k \), conditional on the information set at date \( t \), are given by:

\[
\mu_{x,t+k} = E_t[x_{t+k}] = M \left[ (I_n + \Phi_1 + \cdots + \Phi_{k-1}) \Phi_0 + \Phi_k Y_t \right],
\]

\[
\Sigma_{xx} = V_t[x_{t+k}] = M (I_n + \Phi_1 + \cdots + \Phi_{k-1}) \Sigma (I_n + \Phi_1 + \cdots + \Phi_{k-1})' M'.
\]

Finally, if we define annualized cumulative log-returns as \( x_{t:t+k}^{(k)} = \frac{1}{k} \sum_{i=1}^{k} x_{t+i} \), we obtain that expected excess log-returns and the covariance matrix between \( t \) and \( t+k \), conditional on the information set at date \( t \), are given by:

\[
\mu_{x,t:t+k}^{(k)} = \frac{1}{k} M [(I_n - \Phi_1)^{-1} (k I_n - \Phi_1 - \cdots - \Phi_k) \Phi_0 + (\Phi_1 + \cdots + \Phi_k) Y_t],
\]

\[
\Sigma_{xx}^{(k)} = \frac{1}{k} \left[ M \Sigma M' + M (I_n + \Phi_1) \Sigma (I_n + \Phi_1) Y_t \right] + \cdots + M (I_n + \Phi_1 + \cdots + \Phi_{k-1}) \Sigma (I_n + \Phi_1 + \cdots + \Phi_{k-1})' M'.
\]

These two elements will be used later as we determine the optimal mean-variance assets-only allocation.

---

5 As \( Y_t \) includes inflation and exchange rates, this relation can be used to define nominal or real returns and returns hedged or unhedged against currency risk. One just needs to adjust matrix \( M \) accordingly.
2.3 Modeling Liabilities

Modeling liabilities is clearly a difficult task because the functioning of pension funds is country specific and even often pension-fund specific. Different systems coexist across countries, such as “pay as you go” versus “funded pension plan”, or “defined-contributions” versus “defined-benefits” plans. Depending on the legislation, computing liabilities will be based on different principles and rules. However, this is the road to follow to analyze the real effect of macroeconomic factors on pension funds. Going in that direction requires a description of the liabilities of the pension funds. This includes the dynamics of the population of insured employees and pensioners as well as of the financial cash flows (contributions and benefits).

We adopt the point of view of a Swiss pension fund. The Swiss social security system is based on three pillars. The first pillar (state pension) is designed to cover basic needs. The second pillar (fully-funded occupational pension funds) covers all salaried employees with a minimum annual income. The third pillar is private saving. See Appendix 1 for a more detailed description of the Swiss social security system.

The second pillar relies on pension funds, which can be of different types. They can be regulated by public or private law; they can be single funds or collective foundations; they can be autonomous pension providers or can reinsure part of their risks; they can be based on defined-contributions plans or defined-benefits plans. As most pension funds are private funds with defined-contribution plans (97% of funds, 90% of active members in 2011), we focus on defined contributions regulation. A Master’s Thesis by Barenco (2012) provides more details on the Swiss pension fund industry.6

As the liability of a pension fund is the present value of its expected future cash flows, we need to forecast the future evolution of the pension fund. As discussed in Winklevoss

---

6In 2011, there are 2191 registered pension funds in Switzerland, with 3.787 million active members and 1.041 million beneficiaries. 2099 pension funds are under private regulation (3.161 million active members), whereas 92 pension funds are under public regulation (0.6 million active members). Total liabilities amount to 625 billion CHF. Total benefits paid amount to 44.2 billion CHF and total contributions to 47.1 billion CHF (20 billion paid by employees and 27.1 paid by employers). See Bundesamt für Statistik (2013).
(1993) and Impavido (2011), there exist many ways to generate the future evolution of a pension fund. Regarding the dynamics of the pension-fund population there are two main approaches. In an open pension fund, current plan members will eventually leave, retire, or die and new individuals will join the plan (open-group valuation). In a closed pension fund, members are limited to the current ones (closed-group valuation). As we are interested in long-run investment strategies, we consider an open pension fund, in which the evolution of the plan’s population will be controlled by a replacement rate.\(^7\)

Berkelaar and Kouvenberg (2010) consider the case of a closed-group valuation in which only pension entitlements that have been earned by employees up to the valuation date are considered. As a consequence, they find a relatively low duration of the liabilities, close to 15 years. In a going concern valuation, duration is much longer, in particular in a low interest-rate environment, in which cash flows far in the future contribute in a non-negligible way to liabilities. In our estimates, we find a duration of 24 years in a closed-group valuation, but a duration of more than 50 years in an open group with a replacement rate equal to 1. As we will see below, this has important implications for the investment strategy of the pension fund.

We now discuss the two main ingredients required to generate scenarios of the liability side of a pension fund namely the demographics and the financial cash flows. The dynamics of population and cash flows of a closed pension fund can be described as Markov processes (see Janssen, 1966, Janssen and Manca, 1967, Wolthuis, 1994, and Mettler, 2005, for Switzerland).

### 2.3.1 Population Dynamics

Assume that the initial population is partitioned as follows, according to the status and age of the beneficiaries:\(^8\)

\(^7\)More formally, we adopt a going concern valuation, so that liabilities today are not only based on accrued benefits as of today, but also on future entitlements that will be earned by plan members in the future. In a plan termination valuation, one would assume that the plan is closed at the day of the valuation, so that it does not take future income and benefits into account.

\(^8\)Young active employees start contributing to their pension plan at age 25 and legal retirement age is 65. We also assume that all insured members (or their surviving spouses) die at maximal age 105.
• Active employees (A) with initial population \( P_{A,0} = (P_{A,0}^{(25)}, \ldots, P_{A,0}^{(64)})' \),

• Retired (R) with initial population \( P_{R,0} = (P_{R,0}^{(65)}, \ldots, P_{R,0}^{(105)})' \),

• Dead (M) with initial population \( P_{M,0} = (P_{M,0}^{(25)}, \ldots, P_{M,0}^{(105)})' \).

The initial population of the pension fund is grouped in vector \( P_0 = (P_{A,0}, P_{R,0}, P_{M,0})' \).

In a closed pension fund, with time independent transition probabilities, population dynamics is simply described by \( P_{t+1} = \Pi' P_t \), where \( \Pi \) is a time-homogenous transition matrix:

\[
\Pi = \begin{pmatrix}
\Pi_{AA} & \Pi_{AR} & \Pi_{AM} \\
0 & \Pi_{RR} & \Pi_{RM} \\
0 & 0 & I
\end{pmatrix},
\]

where \( \Pi_{XY} \) is the transition probability from state \( X \) to state \( Y \) and \( I \) is an appropriately-defined identity matrix. Matrix \( \Pi \) is parameterized using mortality tables. As time goes by, active members retire and eventually die.

In practice, most pension funds are open, so that some active members leave the plan before retirement, whereas some new active members join.\(^9\) In this setting, it suffices to increase/decrease the positions in \( P_{A,t} \) where exits or new entries occur. Assume that the active members who leave the plan between date \( t \) and \( t+1 \) are replaced with a ratio \( \psi \). For instance, a ratio \( \psi = 1 \) means that each year all active members leaving the plan will be replaced by a new cohort. We call \( \psi \) the replacement rate. In addition, assume that the age structure of the newly hired employees is denoted by \( \Theta_A = (\Theta_A^{(25)}, \ldots, \Theta_A^{(64)})' \), where \( \Theta_A^{(i)} \) is the proportion of new employees with age \( i \). Then the number of new employees who have to be recruited at date \( t+1 \) to compensate leaving employees and satisfy the assumed replacement rate is: \( [\psi' e' - e' \Pi_{AA}] P_{A,t} \). Finally, given the age structure of the

\(^9\)In Switzerland, plan members who leave the plan before retirement are entitled to vested benefits. As they join a new pension fund, the former one must transfer all vested benefits to the new one. In counterpart, new plan members will bring their vested benefits to the pension fund. Therefore, in such a situation, we reduce pension-fund liability by the value of vested benefits paid to the former employee and increase its liability by the value of vested benefits brought by the new one.
newly hired employees, the new active population at $t + 1$ will be:\(^{10}\)

$$P_{A,t+1} = \Pi'_{AA} P_{A,t} + \Theta_A[\psi' e' - e'\Pi'_{AA}] P_{A,t}, \quad (12)$$

where $e$ is a vector of ones.

### 2.3.2 Financial Cash Flows

Future cash flows include contributions (based on earnings) paid by active members and benefits (based on accumulated saving) paid to pensioners.

Salary received during the period $t$ to $t+1$ has two components: (1) the wage inflation, which is the increase in wage that affects all employees ($\pi^w_{t+1}$); and (2) the merit scale ($ms^{(x,x+1)}$), which depends on the working age of the employees. For active employees of age $x$ at date $t$, the next year salary is given by:

$$S^{(x+1)}_{A,x+1} = S^{(x)}_{A,x} (1 + \pi^w_{t+1}) (1 + ms^{(x,x+1)}) = S^{(x)}_{A,t} (1 + g^{(x,x+1)}_{t+1}), \quad (13)$$

where $g^{(x,x+1)}_{t+1}$ denotes the total salary growth for employees of age $x$.

Accumulated saving has two components: (1) the initial saving accumulated before the employee joins the pension plan; (2) the annual contributions paid by employees to the plan. Both components will generate interest income. For ease of exposition, we assume that the pension fund starts at date 0 with active employees and pensioners having some initial saving.

Initial saving, denoted by $B^{exo}_0 = (B^{exo}_{A,0}, B^{exo}_{R,0}, 0)'$, is exogenous from the pension-fund perspective.$^{11}$ It generates interest income at rate $R^{e}_{t+1}$ for employees and at rate $R^{p}_{t+1}$

---

\(^{10}\)A similar expression may be found in Mettler (2005).

\(^{11}\)In practice, we need to reconstruct this initial component.
for pensioners: \(^{12}\)

\[
\begin{align*}
B_{A,t+1}^{(x+1)exo} &= B_{A,t}^{(x)exo} (1 + R_{t+1}^e), \quad x + 1 = 26, \ldots, 64, \\
B_{R,t+1}^{(x+1)exo} &= B_{R,t}^{(x)exo} (1 + R_{t+1}^p), \quad x + 1 = 66, \ldots, 105.
\end{align*}
\]

The annual contribution paid by employees to the plan is a fraction of current salary and depends on the age of the employee. Once paid, the contribution also earns interest (at rate \(R_{t+1}^e\)). This component is endogenous from the pension-fund perspective, as the fund may decide to change the contribution rate. Starting with \(B_{A,0}^{(x)end} = B_{R,0}^{(x)end} = 0\), the dynamics of endogenous saving is given by:

\[
\begin{align*}
B_{A,t+1}^{(x+1)end} &= B_{A,t}^{(x)end} (1 + R_{t+1}^e) + C_A(x) S_{A,t}^{(x)}, \quad x + 1 = 26, \ldots, 64, \\
B_{R,t+1}^{(x+1)end} &= B_{R,t}^{(x)end} (1 + R_{t+1}^p) + C_A(64) S_{A,t}^{(64)}, \\
B_{R,t+1}^{(x+1)end} &= B_{R,t}^{(x)end} (1 + R_{t+1}^p), \quad x + 1 = 66, \ldots, 105,
\end{align*}
\]

Total saving is then given by: \(B_{A,t+1}^{(x+1)} = B_{A,t+1}^{(x+1)exo} + B_{A,t+1}^{(x+1)end}\) for employees and \(B_{R,t+1}^{(x+1)} = B_{R,t+1}^{(x+1)exo} + B_{R,t+1}^{(x+1)end}\) for pensioners.

Finally, in a defined-contributions plan, financial cash flows received and paid by the pension fund are defined as follows. Contributions paid by employees are a fraction of annual wage. Benefits received by retirees are a fraction of the accumulated saving at time of retirement. Therefore, cash flows combine the evolution of the insured population,

\[^{12}\text{In Switzerland, two different rates are used to remunerate employees and pensioners: } R_{t+1}^e \text{ is close to the LPP/BVG-minimum interest rate and } R_{t+1}^p \text{ is close to the technical rate. The reference base for the LPP/BVG-minimum interest rate is the yield on federal bonds and equity, bond and real estate trends. It was set equal to 1.5\% in 2013 by the Federal Council. The technical rate is determined by pension funds based on their balance-sheet situation and financial markets prospects. The recommendation of the Swiss Chamber of Pension Actuaries for 2013 was 3\%. In general, we have } R_{t+1}^e < R_{t+1}^p. \text{ We use capital } R \text{ for simple return and lower case } r \text{ for log-return.}\]
salaries, and accumulated saving:

\[
CT_{A,t+1}^{(x+1)} = \Pi_{A,t}^{(x,x+1)} P_{A,t}^{(x)} C_{A,t}^{(x)} S_{A,t}^{(x)}, \quad x + 1 = 26, \ldots, 64
\]

\[
CT_{R,t+1}^{(65)} = \Pi_{R,t}^{(64,65)} P_{R,t}^{(64)} C_{R,t}^{(64)} S_{R,t}^{(64)},
\]

\[
CT_{R,t+1}^{(x+1)} = \Pi_{R,t}^{(x,x+1)} P_{R,t}^{(x)} C_{R,t}^{(x)} B_{R,t}^{(x)}, \quad x + 1 = 66, \ldots, 105,
\]

where \( C_{A,t}^{(x)} \) is the contribution rate (in proportion of salary) paid by employee of age \( x \) and \( C_{R,t} \) is the conversion rate (in proportion of accumulated saving).\(^{13}\)

It is clear from these formulas that computing liabilities for active members requires salary-growth forecasts to accrue additional benefits before retirement. These projections are provided by the macro-finance model. We also use Swiss-level statistics on the salary scale across ages, in order to estimate as precisely as possible total salary growth, \( g_{t+1}^{(x,x+1)} \).

For the purpose of exposition, we have presented a very simplified version of the actual dynamics of cash flows of a pension fund.\(^{14}\) We have voluntarily omitted disabled employees, surviving spouses and orphans, exiting employees, and vesting and we did not differentiate male and female populations. The description of the complete model would exceed the usual format of a paper. We use, however, the full-fledged model later on in the empirical section. Use of this full-fledged model shows that, from an economic point of view, differences with this simplified model are of second order.

### 2.3.3 Liabilities

For an asset-liability management exercise, we need to estimate the current and future liabilities to determine the expected return on liabilities. We turn to this issue now. Current time is \( t \) and we consider a time horizon \( T \), for which we need to determine the liabilities. We denote the future liabilities, at time \( t + T \), by \( L_{t+T} \). Once future cash flows have been determined, \( L_{t+T} \) is the present value of expected future cash flows (\( CF_{t+T+i} \))

---

\(^{13}\) Contribution rates and conversion rate are subject to regular changes in Switzerland. In 2011, the contribution rates were equal to 7%, 10%, 15%, and 18% of insured salary for ages [25-34], [35-44], [45-54], [55-65], respectively. The conversion rate was equal to 6.85% for men and 6.8% for women to total accumulated saving.

\(^{14}\) A more complete presentation can be found in Mettler (2005).
discounted with discount rate of the maturity $i$ $(R_{t+T}^{(i)})$:

$$L_{t+T} = E_t \sum_{i=1}^{\infty} \frac{CF_{t+T+i}}{(1 + R_{t+T}^{(i)} )^i}$$  \hspace{1cm} (14)

where $CF_t = \sum_{x=65}^{105} CF_{R,t}^{(x)} - \sum_{x=25}^{64} CF_{A,t}^{(x)}$, i.e., the net cash flow remaining after subtracting contributions paid by employees from benefits paid to pensioners.

An important issue is the choice of the discount rate in expression (14). In actuarial practice, pension funds use the so-called technical rate to discount cash flows. The technical rate is typically based on an average of the long-term government bond rate and the expected return of a risky portfolio, smoothed over a long period of time. See technical guidelines FRP4/DTA4 (Swiss Chamber of Pension Actuaries, 2010). This definition has important consequences: First, as technical rates may be different from one pension fund to the other, comparability of the resulting actuarial balance sheets is questionable. Second, the technical rate is a backward-looking rate and may not reflect the expected evolution of future interest rates and financial returns. This is particularly relevant after the subprime crisis, because the current low level of interest rate has never been seen before.

To avoid these possible biases, we do not follow this approach in the sequel and consider an economic approach, in which cash flows are discounted using term-structure forecasts. To do so, we use a model describing and forecasting the Swiss Confederation term structure. Our approach is based on Nelson and Siegel (1987). We model the dynamics of interest rates for three maturities (3 months, 2 years, and 10 years). With their forecast at date $t+T$, we back out Nelson-Siegel parameters, which describe the level, slope, and curvature of a given term structure. Then, we determine the complete term structure of the Swiss Confederation, $R_{t+T}^{(i)SC}$, at date $t + T$ using Nelson-Siegel formula. See Appendix 2 for more details. This approach ensures that the curve goes through the three forecasted maturities. As pension-fund liability is likely to be riskier than Swiss
Confederation debt, we add a premium $\pi$ to the forecasted rate and use $R_{t+T}^{(i)} = R_{t+T}^{(i)SC} + \pi$ to discount future cash flow $CF_{t+T+i}$.\(^{15}\)

It should be mentioned that it is common in the academic literature to proxy return on liabilities with a long-term government rate, with a maturity typically between 10 and 20 years (Hoevenaars et al., 2008, Van Binsbergen and Brandt, 2008). Such an approach is much simpler to implement, as it does not require modeling the liabilities side of the pension-fund balance sheet. However, it also neglects some potentially important risk factors such as price or wage inflation. More importantly, it dramatically underestimates the actual duration of liabilities. As a consequence, it generally over-estimates the hedging properties of government bonds with 10 to 20-year maturities. Finally, as we will see below, it also over-estimates the actual return on liabilities. For all these reasons, it will lead to an inadequate asset allocation.

Eventually, we define the log-return on liabilities as $r_{L,t+1} = \log(L_{t+1}/L_t)$ and the excess log-return $x_{L,t+1} = r_{L,t+1} - r_{t+1}^{(3m)}$. We also define the expected excess log-return as $\mu_{L,t+1} = E_t[x_{L,t+1}]$ and the variance of the return on liabilities as $\sigma^2_L = V_t[x_{L,t+1}]$. Given the way liabilities are constructed, the return on liabilities is clearly a nonlinear function of the macro-finance factors. The nonlinearity mostly comes from capitalizing interest income and discounting cash flows. Consequently, we cannot use the standard formula described above for a VAR model to compute the expected return and the covariance matrix of assets and liabilities. Instead we rely on Monte-Carlo simulations and proceed as follows. We simulate a large number of samples of the macro-finance model and deduce returns on assets and liabilities. Then, for a given investment horizon, we compute the resulting covariance matrix, which in turn is used for the allocation process.

\(^{15}\)Our approach is different from the one described by Diebold and Li (2006), who forecast Nelson-Siegel parameters. As we have estimated a macro-finance model, it is more natural to forecast directly the reference interest rates and then to recover Nelson-Siegel parameters.
2.4 Portfolio Allocation

We now turn to the optimal portfolio allocation in presence of liabilities. A first intuitive approach consists in matching liabilities cash flows with assets cash flows of the same maturity. This \textit{cash-flow matching}) approach is essentially done using fixed-income instruments such as zero-coupon bonds or inflation-indexed zero-coupon bonds. When such instruments do not exist, a less precise approach relies on \textit{immunization}, which allows an imperfect matching based on equalizing assets and liabilities durations. A limitation of these approaches is their low expected return, as most of the assets held are fixed-income instruments.

A second approach, called \textit{surplus maximization}, drops the requirement of an asset portfolio perfectly correlated with liabilities, and seeks maximal expected return on surplus, defined as the difference between expected returns on assets and on liabilities (Sharpe and Tint, 1990, and Ezra, 1991). By allowing for a mix of asset classes, this approach is expected to produce higher expected returns. In addition, it may hedge non-interest risks borne by pension-fund liabilities. For instance, in the context of portfolio choice, Benzoni, Collin-Dufresne, and Goldstein (2007) have shown that labor income risk can be hedged by investing more in stocks because wages and stocks are highly correlated in the long run. In our context, a pension-fund future stream of cash flows depends on wages. Therefore, the same intuition as for portfolio choice carries over to surplus maximization.

For a formal discussion of surplus maximization, we assume that the pension fund maximizes the funded ratio over the next \(k\) periods. It is defined as \(F_{t+k} = A_{t+k}/L_{t+k}\), with \(A_{t+k} = A_t \exp(r_{A,t:t+k})\) and \(L_{t+k} = L_t \exp(r_{L,t:t+k})\), where \(r_{x,t:t+k}\) denotes the cumulative log return between \(t\) and \(t+k\). In logarithmic scale, the funded ratio is written as \(\log(F_{t+k}) = \log(A_{t+k}) - \log(L_{t+k})\), so that maximizing \(F_{t+k}\) is equivalent to maximizing the surplus log-return \(r_{F,t:t+k} = r_{A,t:t+k} - r_{L,t:t+k}\). We will use the exponent \(\{k\}\) to denote the annualized cumulated return over \(k\) periods. For instance, we have \(r_{F,t:t+k}^{(k)} = \frac{1}{k} \sum_{i=1}^{k} r_{F,t+i} = \frac{1}{k} r_{F,t:t+k}\). We note that, as surplus log-return is defined as dif-
ference, using nominal or real returns will not make a big difference. The presentation below is made with nominal returns, but the same formulas would apply for real returns.

Assuming that the return on liabilities is given, the optimal allocation for horizon $k$ rests on selecting the weights of the risky assets, which we denote by $\alpha_t^{(k)} = \{\alpha_{1,t}^{(k)}, \ldots, \alpha_{N,t}^{(k)}\}$ and the weight of the one-period bond $\alpha_0^{(k)} = 1 - \alpha_t^{(k)} e$, where $e$ denotes a vector of 1. We do not assume any restrictions on portfolio weights for the moment, but later we will also consider the case of positivity constraints $0 \leq \alpha_i^{(k)} \leq 1, i = 0, \ldots, N$. Campbell and Viceira (1999, 2001, 2002) show that the asset portfolio excess log-return can be written as follows, after log-linearization:

$$x^{(k)}_{A,t:t+k} = r^{(k)}_{A,t:t+k} - r^{(3m)(k)}_{t:t+k} = \alpha_t^{(k)} x^{(k)}_{t:t+k} + \frac{1}{2} \alpha_t^{(k)'} \left( \sigma_x^{(k)2} - \Sigma_{xx}^{(k)} \alpha_t^{(k)} \right), \quad (15)$$

where $\Sigma_{xx}^{(k)} = V_t[x^{(k)}_{t:t+k}]$ and $\sigma_x^{(k)2} = \text{diag}(\Sigma_{xx}^{(k)})$, as described in Section 2.2. As a consequence, the surplus annualized log-return is given by:

$$r^{(k)}_{F,t:t+k} = \alpha_t^{(k)'} x^{(k)}_{t:t+k} - x^{(k)}_{L,t:t+k} + \frac{1}{2} \alpha_t^{(k)'} \left( \sigma_x^{(k)2} - \Sigma_{xx}^{(k)} \alpha_t^{(k)} \right), \quad (16)$$

with expected return and variance given by:

$$E_t[r^{(k)}_{F,t:t+k}] = \alpha_t^{(k)'} \mu_t^{(k)} - \mu_{L,t:t+k}^{(k)} + \frac{1}{2} \alpha_t^{(k)'} \left( \sigma_x^{(k)2} - \Sigma_{xx}^{(k)} \alpha_t^{(k)} \right),$$

$$V_t[r^{(k)}_{F,t:t+k}] = \alpha_t^{(k)'} \Sigma_{xx}^{(k)} \alpha_t^{(k)} + \sigma_L^{(k)2} \sigma_{xL}^{(k)},$$

where $\mu_{x,t:t+k}^{(k)}$ denotes the vector of expected excess log-returns on assets, $\sigma_L^{(k)2} = V_t[x^{(k)}_{L,t:t+k}]$ and $\sigma_{xL}^{(k)} = \text{cov}_t[x^{(k)}_{x,t:t+k}, x^{(k)}_{L,t:t+k}]$.

The pension fund maximizes the mean-variance criterion for the surplus:

$$\max_{\{\alpha\}} \quad q_t^{(k)}(\alpha) = \left[ E_t[r^{(k)}_{F,t:t+k}] + \frac{1}{2} V_t[r^{(k)}_{F,t:t+k}] \right] - \frac{\lambda}{2} V_t[r^{(k)}_{F,t:t+k}] \quad (17)$$

$$= \alpha^{(k)'} \mu_{x,t:t+k}^{(k)} - \mu_{L,t:t+k}^{(k)} + \frac{1}{2} \alpha^{(k)'} \sigma_x^{(k)2} - \frac{\lambda}{2} \alpha^{(k)'} \Sigma^{(k)} \alpha + \frac{1}{2} - \frac{\lambda}{2} \left( \sigma_L^{(k)2} - 2 \alpha^{(k)'} \sigma_{xL}^{(k)} \right),$$

20
where $\lambda$ denotes the risk aversion parameter. For pension funds, $\lambda$ is typically much larger than 1. In the absence of portfolio weight restrictions, we find the optimal weights in risky assets:

$$
\alpha_{AL,t}^{(k)} = \frac{1}{\lambda} (\Sigma_{xx}^{(k)})^{-1} \left( \mu_{x,t:t+k}^{(k)} + \frac{1}{2} \sigma_{x}^{(k)2} + (\lambda - 1) \sigma_{xL}^{(k)} \right).
$$

(18)

For comparison purpose, the unrestricted optimal weights in risky assets in an assets-only allocation are given by the following expression (Campbell and Viceira, 2004):

$$
\alpha_{AO,t}^{(k)} = \frac{1}{\lambda} (\Sigma_{xx}^{(k)})^{-1} \left( \mu_{x,t:t+k}^{(k)} + \frac{1}{2} \sigma_{x}^{(k)2} + (\lambda - 1) (-\sigma_{x0}^{(k)}) \right),
$$

(19)

where $\sigma_{x0}^{(k)} = \text{cov}_{t}^{}[x_{t:t+k}^{(k)}, r_{t:t+k}^{(3m)k}]$ is the covariance between asset excess returns and the T-bills return.

Expression (18) shows that the optimal portfolio is composed of three subportfolios:

- The liabilities-hedging portfolio (LHP):

$$
\alpha_{LHP,t}^{(k)} = \left( 1 - \frac{1}{\lambda} \right) (\Sigma_{xx}^{(k)})^{-1} \sigma_{xL}^{(k)}.
$$

(20)

This expression is reminiscent of the OLS regression and corresponds to a projection of liability returns on assets returns. It is therefore the risky portfolio, whose return is the most correlated with the return on liabilities. It is in this sense that this portfolio can be named LHP. This portfolio is independent from expected returns and corresponds to the Global Minimum Variance portfolio in an assets-only allocation.

- The performance-seeking portfolio (PSP):

$$
\alpha_{PSP,t}^{(k)} = \frac{1}{\lambda} (\Sigma_{xx}^{(k)})^{-1} \left( \mu_{x,t:t+k}^{(k)} + \frac{1}{2} \sigma_{x}^{(k)2} \right).
$$

(21)
This portfolio is independent from the characteristics of the pension fund and corresponds to the standard mean-variance portfolio in the context of an assets-only allocation.

- The 3-month T-bills component: $\alpha_{0,t}^{(k)} = 1 - \alpha_{AL,t}^{(k)'}e$.

The sum of LHP weights is not a priori equal to 0, but weights can be positive as well as negative. Imposing positivity restrictions would clearly affect the properties of this portfolio. In particular, it is likely that hedging liabilities risk requires large weights in bonds, so that the amount of wealth devoted to the PSP is likely to be limited. In addition, with positivity restrictions, there is no closed-form solution to the optimal portfolio and portfolio weights must be computed numerically. In the empirical application, we consider two cases: (1) the case with no weights restrictions. (2) the case with positivity restrictions on all asset classes.

It should be mentioned that another, related, approach, called liabilities driven investment (LDI), also relies on the construction of separate portfolios (Martellini, 2006): a first portfolio is explicitly constructed to hedge pension-fund liabilities (liability-matching portfolio); the second portfolio aims at generating performance, following standard asset allocation approaches. Typically, the hedging portfolio would be based on a short position in cash and a long position in long-term bonds or swaps. The consideration of the two types of intermediate portfolios constituting the eventual optimal portfolio appears relevant because it shed insights in the objectives a fund manager has: performance seeking versus liability-hedging.

Obviously, if regulation limits the asset classes or the amounts invested into them, the optimal hedging portfolio may not be feasible.
3 Data and Preliminary Analysis

3.1 Data

The main objective of the paper is to investigate the investment performance of a representative defined-contributions pension fund under Swiss regulation and several configurations regarding the population dynamics, the discount rate, and the weight restrictions. Different types of data are needed. Regarding the macro-finance model, we consider a multi-country framework, composed of the U.S.A., the Euro Area, and Switzerland, so that pension funds can diversify their portfolio internationally. The estimation is performed at quarterly frequency over the period 1985:Q1 – 2013:Q2. The model could be re-estimated each quarter, as new observations become available.

Macroeconomic and financial variables are: output gap, GDP deflator inflation, consumer price inflation, wage inflation, employment growth, unemployment rate, three-month T-bills rate, 10-year government bond rate, real stock return, and log dividend price ratio. Currencies (Euro-Dollar and Swiss Franc-Euro) allow closing the model internationally.

As for standard asset classes, we consider cash, bonds, and stocks in the three zones. We also model the complete Swiss term structure, which is used for discounting pension-fund future cash flows. As for alternative asset classes, we consider commodities and Swiss real estate.\(^{16}\)

Regarding the liabilities model, we use aggregate Swiss data, in order to reflect average population structure, salary scale, etc. We consider a pension fund that would have a given fraction \(1/10^{th}\) of resident population, for both active members and beneficiaries. Assumptions regarding population dynamics, wages, and savings, correspond to Swiss

\(^{16}\)We use the composite S&P Goldman Sachs Commodity index and the IAZI Investment Real Estate index, respectively. As pension funds are often investing in direct real estate projects, we consider a representative index of the Swiss real estate. The IAZI index is based on a pool of approximately 50% of all transactions at actual market conditions. The issue of illiquidity of real-estate investment is mitigated by the long investment horizon we consider.
aggregate data. Assumptions on contribution and conversion rates correspond to the state pension plan (Law on occupational pension schemes, LPP/BVG).

We investigate some properties of returns on assets and liabilities. We start with details on the different steps of the construction of the return on liabilities. Then we move to the hedging properties of the various assets against inflation and real liabilities risks.

### 3.2 Liabilities

In this section, we provide some insight on the liabilities model. The first two ingredients of liabilities are the dynamics of population and cash flows, which determine the numerator of liabilities in equation (14). Both dynamics are affected by the replacement rate, whereas the risk premium determines the denominator of liabilities. Figures 3 to 5 display how the population and cash flows of the representative pension fund are expected to evolve over time, starting with the current structure of the Swiss population and given some assumption for the replacement rate. In Figure 3, the replacement rate is \( \psi = 1 \), i.e., the number of employees does not vary through time (380’000). Employees leaving the plan before retirement are all replaced by new employees, with the same age structure. The number of pensioners (retired, surviving spouses, and disabled) increases over time as employees retire or die, from 101’000 on 2014 to 384’000 in 2054. The number of disabled is small and does not increase over time, so that disabled do not contribute significantly to the population dynamics. The ratio of beneficiaries-over-employees increases from 26% in 2014 to 101% in 2054.

Figure 3 also displays the resulting evolution of cash flows paid/received by the various groups over time. Contributions from employees are expected to increase slowly because of the trend in price and salary (from 4.7 billion CHF per year in 2014 to 8.8 billion in 2054). In contrast, benefits paid to retired and surviving spouses would raise much faster, due to the increase in the number of pensioners and the effect of inflation on future benefits (from −2.4 billion CHF in 2014 to −19.2 billion in 2054). Given that the figure
represents signed cash flows, cash flows paid to beneficiaries are negative and decreasing. The net cash flow of the fund is first positive then negative from 2020 on (from 2.3 billion CHF per year in 2014 to −10.4 billion in 2054).

Figure 4 illustrates the case of a mature pension fund, with a replacement rate of ψ = 0.8, i.e., only 80% of employees leaving the plan are replaced by new employees. Thus, the number of employees decreases by approximately 1.15% per year. At the end of 2054, the number of employees is decreased to 233’000, while the number of beneficiaries has increased to 318’000. In 2054, the total number of insured people is 551’000 and the ratio of beneficiaries-over-employees is as high as 136%. The contributions paid by employees remain at the same level over time (from 4.7 to 5.1 billion CHF per year), as the decrease in population is compensated by salary growth. Benefits increase over time, but less than for ψ = 1 (−16 billion CHF in 2054), so that net cash flows are equal to −10.9 billion in 2054. This amount is slightly larger in absolute value than for ψ = 1.

Figure 5 corresponds to the case of a growing pension fund, with a replacement rate of ψ = 1.2, so that the number of employees increases by approximately 1.15% per year. The number of employees increases to 593’000 in 2054, while the number of beneficiaries also increases, to 466’000, for a ratio beneficiaries-over-employees of 78.6%. As a consequence, the increase in benefits paid to beneficiaries (−23 billion CHF in 2054) is more than compensated by the increase in contributions paid by employees (14.4 billion CHF), so that net cash flows are equal to −8.6 billion CHF in 2054, i.e., less than in the case ψ = 1.

These numbers confirm that it is more difficult to manage a portfolio with a mature population (high beneficiaries-to-employees ratio) than to manage a pension fund with a growing population (low beneficiaries-to-employees ratio). Due to the dynamics of the population, the pension fund will need to generate higher expected returns from the portfolio to fulfill its liability obligations.

To compute liabilities, we discount future expected cash flows using the forecasted term structure of government bond rate plus a risk premium. We consider two risk

---

17In Switzerland, the risk of pension funds is not only borne by the firm sponsor. There is also a guarantee fund, which operates at the country level. It subsidizes institutions with an unfavorable
premia ($\pi = 1\%$ and $2\%$) to evaluate the effect of the discount rate on the optimal allocation in an assets-liabilities framework. In our macro-finance model, interest rates are expected to increase after a minimum level reached in 2013. A premium of $2\%$ approximately corresponds to the average discount rate used by pension funds in 2013 (average technical rate of $3\%$). However, as long-term government rates are expected to increase in the future to a level close to $3.5\%$, adding a premium of $2\%$ would exceed the maximum level of $4.5\%$ recommended by the regulator. We therefore also consider the case $\pi = 1\%$, which would correspond to a discount rate of approximately $4.5\%$ in the long term. To illustrate the dynamics of the discount rate over time, we compute the internal rate of return $\bar{R}_{t+T}$ corresponding to the liability at date $t + T$ by solving the equality:

$$E_t \sum_{i=1}^{\infty} \frac{C_F_{t+T+i}}{(1 + \bar{R}_{t+T})^i} = E_t \sum_{i=1}^{\infty} \frac{C_F_{t+T+i}}{(1 + R^{(0)}_{t+T})^i}.$$ 

This measure is displayed in Figure 6. In the current low interest-rate environment, the long-term government bond rate is expected to start increasing in 2014 up to approximately $3.5\%$ around 2025. The discount rate would reach $4.5\%$ at that time with a premium of $\pi = 1\%$ and $5.5\%$ with a premium of $\pi = 2\%$.

Figure 7 displays the evolution of liabilities of the representative pension fund as measured under our economic approach. It illustrates that the replacement rate and the discount rate may have a huge impact on the present value of future cash flows. Using a low risk premium ($\pi = 1\%$) would generate extreme variations of liabilities in the short run, while high premium ($\pi = 2\%$) would be accompanied by a relatively stable dynamics of liabilities. Sometimes around 2030, the discount rates stabilize. Liabilities stabilize around 2025 before increasing again. The expected value of liabilities is close to 200 billion CHF for low premium ($\pi = 1\%$), whereas it is only 150 billion for a higher premium ($\pi = 2\%$). Changes in replacement rate also plays a role, but less significantly.

age structure and also guarantees statutory benefits owed by insolvent pension funds. However, the guarantee is limited and covers benefits up to a certain upper limit. The guarantee fund is financed by the contributions of all pension funds.
The dynamics of liabilities is driven by a combination of two effects: (1) the large increase in the discount rate; (2) the increase in expected cash flows. To understand why liabilities decrease so much in the short run with a low premium, assume constant cash flows \( CF_t = CF \) and a flat term structure \( R_t^{(i)} = R_t \). In this case, liabilities at date \( t \) would be given by the perpetuity formula, \( L_t = CF/R_t \). Now, if the discount rate increases from 2.5% to 4%, one obtains a decrease in liability by 37.5%. In the case where cash flows are growing with a growth rate of \( g = 1\% \), Gordon’s formula, \( L_t = CF/(R_t - g) \), shows that this time liability decreases by 50%. This is the order of magnitude of the decrease we observe in the figure.

In fact, the main reason for the extreme sensitivity of liabilities to a low discount rate is the fact that the pension fund is not expected to close (open-group approach). This implies that cash flows far in the future still contribute to the liabilities. This effect can be measured by the duration of the liabilities of the fund. The duration depends on the replacement rate and the risk premium, as shown in Table 1. A pension fund with high replacement rate and low premium (\( \psi = 1.2 \) and \( \pi = 1\% \)) has a duration longer than 90 years, while duration decreases to 51 years for low replacement rate (\( \psi = 0.8 \)) and to 42 years when in addition we assume a high premium (\( \pi = 2\% \)).

### 3.3 Expected Returns

In the case of a perpetuity, the return on liabilities would be \( r_{L,t+1} = \log(L_{t+1}/L_t) = \log(R_t) - \log(R_{t+1}) \). It would vary like the negative of the change in the discount rate. Thus, if the term structure would shift upwards, the liabilities would drop and return on liabilities would fall.

Figure 8 displays the term structure of expected returns on liabilities. As interest rates are expected to increase over the coming years, the expected return on liabilities is negative in the short run and increases as horizon increases. With a low premium, the return on liabilities is very low for short horizons. It is as low as \(-25\% \) for a one-year horizon with a replacement rate of \( \psi = 1.2 \) \((-6\% \) with \( \psi = 0.8 \)). In contrast, a
high premium would generate higher, although still negative, return on liabilities (from 
\(-11\%\) with \(\psi = 1.2\) to \(-2.5\%\) with \(\psi = 0.8\)). At a horizon of 20 years, we observe some 
convergence of the average return on liabilities. It lies in the range \([-4.6\%; -1.2\%]\) for 
\(\pi = 1\%\) and \([-0.8\%; 0.1\%]\) for \(\pi = 2\%\). These measures have important implications 
from the pension-fund management perspective. The recent financial crisis has worsened 
the financial situation of most of the pension funds, in particular because of its low return 
environment. Now, the prospects are much more favorable, because the expected increase 
in interest rates will mechanically reduce the expected return on liabilities, although fixed-
income instruments will suffer from the rise in interest rates.

It should be mentioned that this evolution does not only reflect the (past and future) 
evolution of the representative interest rate, but it also captures the expected evolution of 
future cash flows. The figure also displays the 20-year government bond rate, often used 
in the literature to proxy the return on liabilities. As we can see, this return severely 
over-estimates the actual return on liabilities, for two main reasons: first, it does not 
incorporate the effect of the cash flows dynamics. The increase in cash flows contributes 
positively to the value of liabilities, but negatively to the return on liabilities.\(^{18}\) Second, 
the duration of the 20-year bond is much smaller than the duration of liabilities and 
therefore its return is much less sensitive to a translation in the term structure.

**Table 2** displays the expected return and risk of the various asset classes considered in 
our paper. The numbers presented in the table are real annualized log-returns computed 
over a 20-year horizon. As expected, cash has the lowest expected return in the long run. 
Its volatility is far from 0, however, and is actually only slightly below the volatility of 
long-term bonds (3\% versus 4.2\%). Next, in international comparison, U.S. bonds are 
expected to generate higher returns than the European and Swiss counterparts. As foreign 
investments are fully hedged, the difference between expected bond returns mostly comes 
from higher expected U.S. short-term rates. U.S. stock markets should grow at 8\% per 
year which compares to approximately 6.5\% in Europe and 9.3\% in Switzerland. Because

\(^{18}\)In the case of a perpetuity with cash flows growing at rate \(g\), we have \(r_{L,t+1} = -\Delta \log(R_{t+1} - g)\). It 
is easy to verify that the derivative of \(r_{L,t+1}\) with respect to \(g\) is negative.
of stock-return predictability and negative correlation between innovations in dividend-price ratio and equity return, stock volatility is smaller in the long run than in the short run (approximately 20% for a 1-year horizon) (see Barberis, 2000). Commodities expected return is close to 6.5% but with a large risk close to 25%. Swiss real estate promises a return of approximately 3.3% per year with a relatively low long-run volatility.

The expected return on liabilities is equal to $-2.3\%$ in our benchmark case ($\psi = 1$ and $\pi = 1\%$). In a growing plan, the expected return on liabilities will be even lower. In contrast, with a premium of 2%, the expected return would be positive (for $\psi = 1$ and 0.8), although at a relatively low level. Investing in Swiss bond would just match the expected return objective. It is worth emphasizing that the optimal asset allocation will not be fundamentally affected by the level of expected return on liabilities. What will really matter for long-term investment decisions are the expected returns on assets and their hedging properties, i.e., the correlation properties between the return on liabilities and the return on the various asset classes.

We also note that the volatilities of return on liabilities are relatively high, between 8.5% and 19.4%, which would classify liabilities as risky, according to the levels usually observed for assets. The main reason for this relatively high risk is again the duration of the liabilities, because increasing the duration implies a higher sensitivity to changes in interest rates. For instance, for $\pi = 1\%$, increasing the replacement rate from 0.8 to 1.2 implies an increase in duration from 50 to 92 years and an increase in volatility from 10% to 19.4%. A higher discount premium would render liabilities less risky as they would be less affected by changes in interest rates.

### 3.4 Hedging Properties

We now investigate the hedging properties of the various asset classes. We decompose hedging properties into two components: inflation hedging and real-liabilities hedging properties. Inflation hedging is a fundamental property of asset classes, as it is independent from the liabilities of a given pension fund. In contrast, real-liabilities hedging will be
pension-fund specific. As we consider a representative pension fund, we are mostly concerned by the hedging properties against real-liabilities risk as measured in our different scenarios.

Table 3 presents correlations between nominal return and inflation on the one hand and between real return on assets and real return on liabilities on the other hand. As already emphasized by Campbell and Viceira (2005) and Hoevenaars et al. (2008), a short-term bond is the best inflation hedge at long horizon. As the main objective of monetary policy is to control inflation, both series are intrinsically related. For Switzerland, the correlation between the two series is as high as 80%. In contrast, long-term bonds only provide a weak hedge against inflation, with a low 26% correlation. Swiss stocks turn out to be an even worse hedge with a correlation of only 12%. This limited hedging capability was already mentioned by Fama and Schwert (1977) and Campbell and Shiller (1988): high inflation increases interest rates, which lowers stock prices, but it also tends to increase dividends, leading to higher stock prices. Both effects partly compensate each other and the net effect is small.

The second best hedges against inflation are commodities, with a correlation of 65%. This property is expected, given that commodities prices tend to anticipate future changes in consumption prices (Gorton and Rouwenhorst, 2006). In contrast, Swiss real estate does not provide a good hedge against inflation, probably because house prices in Switzerland have grown relatively fast over the last decade, while otherwise inflationary pressures were under control. We also observe that all of our measures of return on liabilities are negatively correlated to inflation, in the range \([-26.6\%; -16.1\%]\). This suggests that the main effect of inflation is through the discount rate. Cash flows are also affected but in a more ambiguous manner, because in the long run, inflation increases both contributions and benefits.

The table shows the hedging properties for real-liabilities risk. Although the ranking of assets in terms of hedging capability is not altered by changing the replacement rate and the premium, we observe that the correlations change significantly from one case to
The best hedges are Swiss long-term bonds, with a correlation ranging from 50% (for $\psi = 1.2$ and $\pi = 1\%$) to 77% (for $\psi = 0.8$ and $\pi = 2\%$). This observation demonstrates that, although the correlation of bonds and liabilities is strong, the relation is far from being perfect and reveals the importance of explicitly modeling the liabilities of a pension fund. There are in fact two main reasons for this result: First, the 10-year government bond is a poor proxy for liabilities because the liabilities duration is much longer than the duration of this bond. Second, there are other sources of risk included in the return on liabilities, which are not captured by long-term bond returns. This includes in particular the dynamics of the pension fund population and the dynamics of salaries. Intuitively, one may have anticipated those findings. It is only a full model as the one considered here that allows us quantifying the actual amounts. The next best hedges are European and U.S. bonds, with rather large correlations (all above 45%).

Equities have intermediate hedging properties. In this class, U.S. equities are better hedges than European and Swiss equities. For low replacement rates and high premium (low duration), the correlations with real liabilities are the highest (36% for U.S. equities and 24% for Swiss equities). For high replacement rate and low premium (long duration), the correlations are relatively low (30% for U.S. equities and only 8% for Swiss equities). Last, we note that Swiss real estate is a relatively good hedge for low replacement rate and low premium (correlation of 37%) but is much less correlated with liabilities for high replacement rate (18%).

Cash and commodities have the worst hedging properties in our asset universe (with correlations in the range $[-55\%; -35\%]$ and $[-30\%; -16\%]$, respectively).

To sum up, high replacement rate and low premium imply higher liabilities duration and in general lower correlation with all asset classes. This result suggests that it is more difficult to find good hedges against real liabilities risk in case of longer duration.

\[\text{Having negative correlation with liabilities does not necessarily exclude these assets from the optimal portfolio. It just suggests that they would not be part of the liabilities-hedging portfolio, if negative weights were precluded. However, these assets could be part of the performance-seeking portfolio, provided their expected returns are large enough or their correlations with other assets provide sufficient diversification.}\]
4 Strategic ALM

In this section, we analyze the implementation of assets-liabilities strategies by pension funds in a low interest rate environment. We measure the cost of several suboptimal strategies. In particular, we investigate the case of assets-only allocation or positivity restrictions. All the allocations we discuss in this section are based on real returns over a 20-year horizon.

Equations (18) and (19) give the unrestricted optimal portfolio weights for an assets-liabilities and an assets-only allocation, respectively. The corresponding liabilities-hedging portfolio (LHP) and optimal global minimum variance portfolio (GMVP) are defined by:

\[ \alpha^{(k)}_{LHP,t} = \left( \Sigma^{(k)}_{xx} \right)^{-1} \sigma^{(k)}_{xL}, \quad \text{and} \quad \alpha^{(k)}_{0,LHP,t} = 1 - \left( \alpha^{(k)}_{LHP,t} \right)' e, \]
\[ \alpha^{(k)}_{GMVP,t} = - \left( \Sigma^{(k)}_{xx} \right)^{-1} \sigma^{(k)}_{x0}, \quad \text{and} \quad \alpha^{(k)}_{0,GMV,t} = 1 - \left( \alpha^{(k)}_{GMV,t} \right)' e. \]

These portfolios may be out of reach because of the negativity of some of the weights.

Table 4 shows the optimal allocation and portfolio performance for the assets-only (AO) and assets-liabilities (AL) portfolios, for our benchmark case (\( \psi = 1 \) and \( \pi = 1\% \)). We consider three different portfolios. The first portfolios are the global minimum variance portfolios (denoted by GMVP for assets-only and LHP for assets-liabilities). Differences between the GMVP and LHP are important because they are unrelated to expected returns, which are known to be difficult to forecast and only consider hedging properties. We also consider mean-variance portfolios with two different levels of risk aversion, \( \lambda = 50 \) and \( 20 \), as described by equation (18) for assets-liabilities and (19) for assets-only. As pension funds are expected to adopt rather safe strategies, we consider the case \( \lambda = 50 \) as our benchmark. It corresponds to an annualized volatility of the asset.

\[ ^{20}\text{As it can be seen in the assets-liabilities optimization program (17), using nominal or real returns will not affect the optimal portfolio significantly, as the criterion is based on the expected return on the asset portfolio in excess of the return on liabilities. The only difference would come from the definition of the covariance matrix for nominal versus real returns. For this reason, to save space, we only report our results in real terms, so that the optimal portfolio is constructed based on asset returns in excess of expected inflation.} \]
portfolio of approximately 3-4%, a level often adopted by insurance companies. The case \( \lambda = 20 \) corresponds to a volatility in the range 5-6% often chosen by pension funds or endowment funds.

### 4.1 Unrestricted Optimal Portfolios

We start with unrestricted optimal portfolios (Panel A) and with assets-only portfolios. If we consider the unrestricted GMVP, it is essentially long in cash and Swiss bonds (53% and 36%, respectively) and short in U.S. bonds (−18%). The large weight in cash is not surprising as cash has low volatility and provides a very good hedge against inflation. If we now consider the mean-variance portfolio with \( \lambda = 50 \) and 20, there is a reallocation in favor of riskier assets. The weights of equities, U.S. bonds, commodities, and real estate increase, whereas the weights of cash and Swiss and European bonds decrease. For \( \lambda = 20 \), the optimal assets-only portfolio is short in cash and Swiss bonds and long in U.S. bonds and equities, Swiss equities, and real estate.

We now turn to assets-liabilities portfolios, which take the correlation of assets with liabilities into account. The unrestricted LHP is very different from the unrestricted GMVP. The investor mostly invests in Swiss, U.S., and European bonds (133%, 55%, and 44%, respectively) and short in cash and real estate (−129% and −24%). We clearly see the effect of the correlation matrix used for the GMVP and LHP and the role of hedging liabilities risk. On the one hand, the GMVP tries to minimize the absolute portfolio risk, by combining short-term and long-term bonds. On the other hand, the LHP tries to minimize the variance of the difference between the return on the asset portfolio and the return on liabilities. This is done by borrowing cash and investing the portfolio in all types of long-term bonds (for a total of 233% of initial wealth).

In the optimal mean-variance portfolio with \( \lambda = 50 \) and 20, the LHP is combined with a performance-seeking portfolio. It therefore reduces the weight devoted to bonds and increases the weights to equities. We note that the role played by U.S. bonds is twofold:
it contributes to the LHP, but its weight also increases when the pension fund put more emphasis on performance.

The table also reports the expected return and risk ($\mu_A$, $\sigma_A$) of the asset portfolio and the expected return and risk ($\mu_S$, $\sigma_S$) of the surplus. Several results are worth mentioning. First in all cases, the optimal allocations are expected to generate positive surplus return: close to 4% for lowest risk portfolios and close to 10% for riskier portfolios. Second, the risk of the surplus is rather high (above 10% for all portfolios). This reflects the high risk of the return on liabilities for low premium. Third, in all cases, the assets-only portfolios generate lower expected return and higher risk for the surplus than the corresponding assets-liabilities portfolios.

To evaluate the gain for an investor of being allowed to switch from a suboptimal portfolio to the optimal assets-liabilities portfolio, we use certainty equivalent. It is defined as the difference between the utility of the optimal assets-liabilities portfolio $q_{t}^{(k)}(\alpha_{AL,t}^{(k)})$ and the utility of the suboptimal portfolio $q_{t}^{(k)}(\alpha_{sub,t}^{(k)})$:

$$CE_{t}^{(k)} = q_{t}^{(k)}(\alpha_{AL,t}^{(k)}) - q_{t}^{(k)}(\alpha_{sub,t}^{(k)})$$

$$= (\alpha_{AL,t}^{(k)} - \alpha_{sub,t}^{(k)})'(\mu_{t,t+k}^{(k)} + \frac{1}{2}\sigma_{x}^{(k)}2) - \frac{1}{2} - \frac{1}{2} (\alpha_{AL,t}^{(k)} - \alpha_{sub,t}^{(k)})'(\sigma_{L}^{(k)}2)$$

As the formula shows, certainty equivalent allows us to convert differences in portfolio variances in units of portfolio expected return. For instance, for $\lambda = 50$, a reduction of 1% of the surplus volatility (from 12% to 11%) is equivalent to an increase in surplus expected return of 5.4%, which reflects the fact that mitigating risks matters a lot for pension funds.

In the present context, we consider the premium that the investor is willing to pay to switch from the assets-only portfolio to the assets-liabilities portfolio. There is a huge gain for pension funds to invest in the assets-liabilities portfolio: the premium is equal to 14% in the benchmark case $\lambda = 50$. It decreases to 5.5% for lower risk aversion ($\lambda = 20$).
For $\lambda = 50$, this premium is due to a combination of two effects: the decrease in surplus volatility (from 13.6% to 11.4%) and the increase in surplus expected return (from 4.1% to 5%).

4.2 Optimal Portfolio with Positivity Restrictions

We have discussed the optimal portfolio allocation when the pension fund is allowed to short some asset classes. This is not the case in practice. We therefore consider now the case with positivity restrictions. In this case, the optimal portfolios for the surplus maximization and the asset only cases are obtained by solving the following programs:

$$\min_{\{\alpha\}} V[r_F^{(k)}] \quad \text{and} \quad \min_{\{\alpha\}} V[r_A^{(k)}],$$

where positivity restrictions are imposed by setting $0 \leq \alpha_i \leq 1$, $i = 1, \cdots, N$, and $1 - \alpha' e \geq 0$ during the numerical optimization. Positivity restrictions will clearly play a key role in the assets-liabilities allocation, as the LHP is naturally constructed on shorting cash.

If we start with assets-only portfolio, we note that there are only limited differences with the unrestricted portfolios discussed before (Panel B). In the GMVP, the portfolio is mostly invested in cash and Swiss bonds (43% and 31%, respectively) with zero weight in U.S. bonds. In the mean-variance portfolio with $\lambda = 50$, we have essentially the same portfolio as before. In fact, the premium the investor would be willing to pay to switch from the restricted assets-only portfolio to the unrestricted one is negligible.

In contrast, the possibility for pension funds to borrow cash for investing in the liabilities-hedging portfolio plays an important role. The reason is that the optimal, unrestricted, LHP is short in cash. When borrowing cash is not allowed, the optimal LHP has a larger weight in Swiss and European bonds and U.S. equities (44%, 47%, and 9%, respectively). This evidence suggests that allowing pension funds to hedge their liabilities through borrowing cash and investing in a diversified bond portfolio would
help enhancing global portfolio return. We observe similar patterns in the mean-variance portfolios with $\lambda = 50$ and $20$. For all assets-liabilities portfolios, imposing positivity restrictions results in smaller expected return and higher risk of the surplus.

The premium the investor would be willing to pay to be allowed to invest in the unrestricted mean-variance portfolio is 7.7% for $\lambda = 50$ and 4.2% for $\lambda = 20$. Such large premiums are due to the benefits for the investor to borrow cash for investing in long-term bonds. We also observe that, when cash borrowing is not allowed, the premium to switch from assets-only to assets-liabilities allocation is decreased: from 14% to 6.5% for $\lambda = 50$ and from 5.5% to 2.1% for $\lambda = 20$. This result is explained by the fact that the pension fund has less degree of freedom when not allowed to borrow cash for hedging its liabilities.

It should be noticed, however, that even with positivity restrictions, the gain of assets-liabilities management is quite substantial compared to standard assets-only allocation. For $\lambda = 50$, the surplus expected return is slightly decreased (from 4.1% to 3.5%) but the surplus volatility is decreased from 13.6% to 12.5%. As we can see from this decomposition, the main implication of investing in the assets-only portfolio is that it does not put sufficient emphasis on hedging liabilities risk and therefore does not invest sufficiently in bonds.

### 4.3 Alternative Specifications

In Table 5, we investigate the effect of changing the replacement rate on the optimal allocation, we remember that this is equivalent to changing liabilities duration. The first three columns of the table are devoted to the case $\psi = 0.8$ (smaller duration) and the last three columns to the case $\psi = 1.2$ (longer duration).

We start again with unrestricted allocations. We note that the main effect of the replacement rate is to change the size of the investment in cash and bonds, which we could interpret as a change in leverage. With $\psi = 0.8$, the weights of Swiss, U.S., and European bonds are 112%, 43%, and 41%, respectively, while the pension fund
should borrow 98% of cash. With $\psi = 1.2$, the weights increase to 186%, 67%, and 31%, respectively, while the pension fund should borrow 162% of cash. In doing so, the investor tries to increase the duration of the LHP. As the optimal allocation of a pension fund with growing population is very short in cash, it is not surprising that the cost of the assets-only allocation is higher than in the case of a mature plan. It is clearly apparent that the lack of longer-term bonds is a strong limitation in the construction of an effective LHP.

If we now consider the portfolios with positivity restrictions, we observe that the optimal weights barely change compared with the case $\psi = 1$. The reason is again that, as cash borrowing is not allowed, the pension fund cannot leverage the portfolio to increase its duration. The table also reveals that for $\psi = 1.2$ the search for long duration assets points in favor of U.S. equities. For the LHP, their weight is as high as 20%, whereas it was only 9% for $\psi = 1$. In the case of a growing plan ($\psi = 1.2$), the premium to pay for switching to the unrestricted portfolio is higher: for $\lambda = 50$, the cost of assets-only allocation is 22% per year and the cost of positive weights restrictions is 13%. This is approximately twice the costs we found for $\psi = 1$. Conversely, for a mature plan, the opportunity costs of the assets-only allocation and positive weights restrictions are lower, although they remain substantial.

Finally, we consider in Table 6 the optimal assets-liabilities allocation in the case of a higher risk premium, $\pi = 2\%$. As discussed before, this case corresponds to liabilities with higher expected returns and lower volatilities. In all cases, the unrestricted LHP is long in bonds and short in cash (for instance, 205% in bonds and $-111\%$ in cash for $\psi = 1$). However, the short position in cash is clearly reduced compared to the low premium case (by approximately 20 to 30%). This can be explained by the fact that the duration is smaller with higher premium and therefore increasing asset portfolio duration by shorting cash is less necessary.

In LHP, we observe that the expected return on surplus is rather low (below 1% for $\psi = 0.8$ with or without weights restrictions), with a relatively high risk (a minimum
of 5%). This suggests that there may not be sufficient room for manoeuvre to generate positive surplus over the long term.

5 Conclusion

There exists a substantial actuarial literature dealing with techniques on how to generate future cash flows for defined-benefits plans. Unfortunately, the literature on how to manage optimally defined-contributions plans is scarce. This is regrettable, since at the time this paper is written, most pension plans have moved to defined contributions. The reason for this is the increase in life expectancy, which puts pressure on the sponsoring companies of many pension funds, facing difficulties in honoring past promises. This problem is exacerbated because of possible erroneous investments during the 2000/03 recession and because of significant losses during the financial crisis associated with the current phase of low interest rates.

In this paper, we show how to perform an ALM study from a financial prospective for defined-contributions plans. Given international interest in the way the Swiss pension fund system operates, we adopt the Swiss framework in our investigation. This framework tells us how to calibrate the model for our empirical investigation.

To perform this ALM study, we start with an economic model that drives both the asset and liability sides of the pension-fund balance sheet. The economic model provides forecasts on the macroeconomic and financial factors that drive expected returns and risks of assets and liabilities. On the asset side, we generate scenarios for the U.S.A., the Euro Area, and Switzerland, which allow us to forecast expected returns and risks of nine asset classes, diversified internationally. On the liability side, we adopt an economic approach in the way liabilities are constructed, so that future financial cash flows are discounted using term-structure forecasts. This approach allows us to estimate the correlation between assets and liabilities, which is the key ingredient in constructing a liabilities hedging portfolio.
We calibrate the economy using quarterly data between 1985:Q1 and 2013:Q2. Using a certainty equivalent approach, we show that failing to construct a liabilities-hedging portfolio results in a cost between 5 and 15% per year. The main reason for such a large cost is that the optimal assets-only portfolio is typically long in cash, whereas hedging liabilities requires the pension fund to be short in cash. It follows that imposing positivity restrictions in the construction of the portfolio also results in a large cost, between 4 and 8% per year. This estimate suggests that allowing pension funds to hedge their liabilities through borrowing cash and investing in a diversified bond portfolio would help enhancing global portfolio return.
References


Appendix

Appendix 1: Swiss Social Security System

Details on the Swiss pension system can be found in Büttler and Ruesch (2007) and Barenco (2012).

The Swiss pension system is based on three pillars: (1) a state pension (first pillar); (2) a fully-funded occupational pension funds (second pillar); (3) a private pension (third pillar).

The state pension system (AHV-IV / AVS-AI, i.e., old-age, survivors, and disability insurance), is a pay-as-you-go-system, introduced in 1948. It is designed to provide a basic subsistence level to all retired residents in Switzerland. Current workers have to finance the pensions of current retired people, through a tax on labor income.

As the first pillar only covers basic needs, to reach a normal life standard (say, 60% of last labor income) a complementary insurance is required. The occupational pension fund system is designed for complementing the first pillar. To account for unfavorable demographic evolution (increase in life expectancy, decrease in birth rate), responsible for the rise in old-age dependency ratio, it became mandatory in 1985 (BVG / LPP, i.e., occupational retirement, survivors and disability pension plans). It covers all salaried employees with a minimum annual income. Together, the first two insurance systems should ensure that retired people maintain their former standard of living, i.e., they should jointly provide approximately 60% of the last salary.

The second pillar works as follows: contributions paid by employees are credited on a personal account. The accumulated capital generates an annual interest, which should be at least equal to the minimum interest rate defined by the federal government (minimum BVG / LPP rate). All pension funds have to cover the mandatory occupation benefit insurance. But most of them also offer extra-benefit insurance. Pension benefits are strictly proportional to the accumulated saving. At retirement, the annual pension is defined as the accumulated saving times a conversion factor. The legal conversion factor
is fixed by the federal government. Pension funds can use a different factor, provided the annual pension is at least equal to the legal minimum pension.

Most occupational pension plans were initially defined as defined-benefit plans. However, to ensure portability of retirement saving, most plans have now switched to defined-contribution plans.

The third pillar is voluntary privately-financed saving. It is designed to cover the income gap during the old age. A first pillar 3a is supported by the federal government through tax exemption and is typically invested in bank and insurance products. Pillar 3b is free private saving, so that it includes all types of savings.

Appendix 2: Forecasting the Term Structure of Interest Rates

A common approach to forecasting the term structure of interest rates is based on Nelson and Siegel (1987). Diebold and Li (2006) are the first to use the Nelson-Siegel approach for forecasting. Their idea is to estimate the three Nelson-Siegel $\beta$ parameters, which describe level, slope, and curvature of the term structure. Then, they use a VAR model to predict the future $\beta$ parameters. Forecasts of $\beta$ parameters are in turn used to forecast the term structure. We experimented with this approach and found that it is not performing well, even in sample. The main reason is that the $\beta$ parameters are extremely persistent and are difficult to predict with past values. As a consequence, the approach fails at predicting changes in the term structure patterns, so that the predicted term structure is often rather far from the actual one.

Our approach is also related to Nelson and Siegel, but in a different way. As we have a model for predicting the main macroeconomic and financial factors, we use our model to obtain reference points along the term structure. In fact, in our VECM approach, we describe the dynamics of the three-month T-bills rate and the 2-year and 10-year government bond rates. Using these points, we then infer the entire term structure.

Formally, let us consider the forecast of the term structure for the next quarter, from $t$ to $t + 1$. The macro-finance model gives us predictions for the three bond rates, which we
denote by $\hat{r}_{t+1}^{(3m)}$, $\hat{y}_{t+1}^{(2)}$, and $\hat{y}_{t+1}^{(10)}$. The specification proposed by Nelson and Siegel (1987) is that the (zero-coupon) yield of maturity $m$ should be driven by the relation:

$$NS(\beta, m) = \beta_1 + \beta_2 \frac{1 - e^{-m/\tau}}{m/\tau} + \beta_3 \left( \frac{1 - e^{-m/\tau}}{m/\tau} - e^{-m/\tau} \right).$$

Limit maturities indicate that the overnight rate should be equal to $NS(\beta, 0) = \beta_1 + \beta_2$, whereas the rate of a perpetuity should be equal to $NS(\beta, \infty) = \beta_1$. These limit cases allow us to interpret $\beta_1$ and $\beta_2$ as the level and the slope of the term structure. Parameter $\tau$ corresponds to the maximum of the curvature of the term structure. Diebold and Li (2006) and Diebold, Rudebusch, and Aruoba (2006) have concluded that $\tau$ should be set such that the maximum of the curvature is reached for a maturity of approximately 2 years. In this context, $\beta_3$ can be interpreted as the curvature of the term structure.

Our approach consists in using our interest rates forecasts to back out the three $\beta$ parameters: $(\hat{r}_{t+1}^{(3m)}, \hat{y}_{t+1}^{(2)}, \hat{y}_{t+1}^{(10)}) \rightarrow (\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$. The expressions for the $\beta$ parameters are cumbersome to obtain, but straightforward. Then, given these parameters $(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$, we can reconstruct the complete term structure, through the expression $NS(\hat{\beta}, m)$.

The main advantage of our approach is that it ensures that the forecasted term structure passes through the forecasts produced by the macro-finance model for the three-month T-bills rate and the 2-year and 10-year government bond rates. The forecasted term structure is therefore consistent with the macro-finance model.
Table 1: Duration of Liabilities (in years)

<table>
<thead>
<tr>
<th>Premium</th>
<th>Replacement rate</th>
<th>$\psi = 0.8$</th>
<th>$\psi = 1.0$</th>
<th>$\psi = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi = 1%$</td>
<td>50.9</td>
<td>70.1</td>
<td>92.4</td>
<td></td>
</tr>
<tr>
<td>$\pi = 2%$</td>
<td>41.6</td>
<td>52.9</td>
<td>66.8</td>
<td></td>
</tr>
</tbody>
</table>

Note: The table reports the duration of liabilities, for various values of the replacement rate ($\psi = 0.8, 1, \text{and} 1.2$) and the risk premium ($\pi = 1\% \text{ and} 2\%$). Duration of liabilities at date $t$ is computed as:

$$D_t = \frac{1}{E_t} \sum_{i=1}^{\infty} i \frac{CF_{t+i}}{(1 + R^{(i)})^i} \quad \text{where} \quad L_t = E_t \sum_{i=1}^{\infty} \frac{CF_{t+i}}{(1 + R^{(i)})^i}.$$
### Table 2: Real Expected Return and Volatility of Assets and Liabilities

<table>
<thead>
<tr>
<th></th>
<th>Expected return</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash</td>
<td>0.34</td>
<td>3.05</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>1.31</td>
<td>4.19</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>7.87</td>
<td>9.43</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>0.67</td>
<td>5.06</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>6.50</td>
<td>11.08</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>0.49</td>
<td>4.23</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>9.35</td>
<td>11.70</td>
</tr>
<tr>
<td>Commodities</td>
<td>6.61</td>
<td>23.12</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>3.31</td>
<td>6.11</td>
</tr>
<tr>
<td><strong>Liabilities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi = 1.0; \pi = 1%$</td>
<td>-2.28</td>
<td>14.47</td>
</tr>
<tr>
<td>$\psi = 1.0; \pi = 2%$</td>
<td>0.19</td>
<td>10.75</td>
</tr>
<tr>
<td>$\psi = 0.8; \pi = 1%$</td>
<td>-0.93</td>
<td>10.20</td>
</tr>
<tr>
<td>$\psi = 0.8; \pi = 2%$</td>
<td>0.46</td>
<td>8.46</td>
</tr>
<tr>
<td>$\psi = 1.2; \pi = 1%$</td>
<td>-4.27</td>
<td>19.38</td>
</tr>
<tr>
<td>$\psi = 1.2; \pi = 2%$</td>
<td>-0.48</td>
<td>13.61</td>
</tr>
</tbody>
</table>

Note: The table reports the expected return and volatility of the asset classes and liabilities we consider in the paper. The measures correspond to annualized log-returns over 20 years in real terms.
Table 3: Hedging Properties of Asset Classes

<table>
<thead>
<tr>
<th></th>
<th>Correlation with inflation (\psi = 1.0)</th>
<th>Correlation with liabilities (\pi = 1%)</th>
<th>Correlation with liabilities (\pi = 2%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
<td>(nominal)</td>
<td>(real)</td>
<td>(real)</td>
</tr>
<tr>
<td>Cash</td>
<td>79.5</td>
<td>-40.2</td>
<td>-51.9</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>44.4</td>
<td>50.5</td>
<td>60.4</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>28.8</td>
<td>29.9</td>
<td>34.5</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>25.3</td>
<td>54.4</td>
<td>67.2</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>10.7</td>
<td>23.2</td>
<td>27.4</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>26.2</td>
<td>58.3</td>
<td>71.9</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>11.8</td>
<td>19.5</td>
<td>24.0</td>
</tr>
<tr>
<td>Commodities</td>
<td>64.8</td>
<td>-23.4</td>
<td>-28.4</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>25.4</td>
<td>27.9</td>
<td>37.3</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td>–</td>
<td>-18.5</td>
<td>-26.2</td>
</tr>
</tbody>
</table>

Note: The first column of the table reports the correlation between nominal returns on assets and liabilities and inflation. Subsequent columns report the correlation between real return on assets and real return on liabilities. The last row reports the correlation between inflation and the various definitions of (nominal) liabilities. Inflation is measured by the growth rate of the consumer price index. We consider an horizon of 20 years.
Table 4: Optimal Assets-Only and Assets-Liabilities Portfolios ($\psi = 1$ and $\pi = 1\%$)

<table>
<thead>
<tr>
<th></th>
<th>Assets-Only</th>
<th>Assets-Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GMVP $\lambda = 50$</td>
<td>LHP $\lambda = 50$</td>
</tr>
<tr>
<td>Cash</td>
<td>53.0</td>
<td>-129.0</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>-17.9</td>
<td>55.3</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>11.1</td>
<td>9.7</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>14.9</td>
<td>44.2</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>-2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>36.4</td>
<td>133.0</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>-3.3</td>
<td>8.3</td>
</tr>
<tr>
<td>Commodities</td>
<td>2.7</td>
<td>-0.2</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>5.0</td>
<td>-24.3</td>
</tr>
</tbody>
</table>

Panel A: No weight restriction

<table>
<thead>
<tr>
<th></th>
<th>Assets-Only</th>
<th>Assets-Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>53.0</td>
<td>-129.0</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>-17.9</td>
<td>55.3</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>11.1</td>
<td>9.7</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>14.9</td>
<td>44.2</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>-2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>36.4</td>
<td>133.0</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>-3.3</td>
<td>8.3</td>
</tr>
<tr>
<td>Commodities</td>
<td>2.7</td>
<td>-0.2</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>5.0</td>
<td>-24.3</td>
</tr>
</tbody>
</table>

Expected return and risk:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_A$</td>
<td>1.1</td>
<td>2.0</td>
<td>1.3</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>2.1</td>
<td>7.9</td>
<td>4.3</td>
</tr>
<tr>
<td>$\mu_S$</td>
<td>3.4</td>
<td>4.3</td>
<td>11.2</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>13.6</td>
<td>11.2</td>
<td>14.4</td>
</tr>
</tbody>
</table>

Cost of Assets-Only allocation

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>-29.2</td>
<td>14.3</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>46.7</td>
<td>38.9</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>44.3</td>
<td>27.6</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>0.0</td>
<td>9.5</td>
</tr>
<tr>
<td>Commodities</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Panel B: Positivity restrictions

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>43.4</td>
<td>0.0</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>6.5</td>
<td>9.0</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>10.1</td>
<td>46.7</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>31.3</td>
<td>44.3</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Commodities</td>
<td>3.7</td>
<td>0.0</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>5.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Expected return and risk:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_A$</td>
<td>1.4</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>2.2</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>$\mu_S$</td>
<td>3.7</td>
<td>3.6</td>
<td>5.6</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>13.5</td>
<td>12.3</td>
<td>13.3</td>
</tr>
</tbody>
</table>

Cost of Assets-Only allocation

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>-14.9</td>
<td>6.5</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>-50.5</td>
<td>21.2</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>-20.4</td>
<td>11.7</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>-10.6</td>
<td>5.6</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>-1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>-30.0</td>
<td>15.0</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>-10.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Commodities</td>
<td>-20.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>-10.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Note: The table reports the optimal portfolio weights corresponding to the assets-only portfolio and assets-liabilities portfolio, for the global minimum variance portfolio (GMVP and LHP, respectively) and for mean-variance portfolios with risk aversion $\lambda = 50$ and $\lambda = 20$. Log-returns are in real terms. $(\mu_A, \sigma_A)$ and $(\mu_S, \sigma_S)$ denote the expected return and volatility of the assets portfolio and the surplus, respectively.
Table 5: Optimal Assets-Liabilities Portfolios ($\psi = 0.8$ and $1.2$)

<table>
<thead>
<tr>
<th></th>
<th>($\psi = 0.8; \pi = 1%$)</th>
<th>($\psi = 1.2; \pi = 1%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LHP $\lambda = 50$</td>
<td>LHP $\lambda = 20$</td>
</tr>
<tr>
<td><strong>Panel A: No weight restriction</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash</td>
<td>-97.6</td>
<td>-128.1</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>43.2</td>
<td>70.2</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>4.5</td>
<td>11.8</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>41.1</td>
<td>32.2</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>4.0</td>
<td>4.7</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>112.0</td>
<td>88.4</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>4.2</td>
<td>17.1</td>
</tr>
<tr>
<td>Commodities</td>
<td>-0.3</td>
<td>4.3</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>-11.0</td>
<td>-0.5</td>
</tr>
<tr>
<td><strong>Expected return and risk:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_A$</td>
<td>1.6</td>
<td>4.4</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>6.7</td>
<td>7.1</td>
</tr>
<tr>
<td>$\mu_S$</td>
<td>2.5</td>
<td>5.3</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>6.5</td>
<td>6.9</td>
</tr>
<tr>
<td>Cost of Assets-Only allocation</td>
<td>20.0</td>
<td>9.8</td>
</tr>
<tr>
<td>Cost of positivity restrictions</td>
<td>7.2</td>
<td>4.6</td>
</tr>
<tr>
<td><strong>Panel B: Positivity restrictions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>5.3</td>
<td>20.3</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>44.8</td>
<td>37.0</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>50.0</td>
<td>33.2</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>0.0</td>
<td>9.5</td>
</tr>
<tr>
<td>Commodities</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>Expected return and risk:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_A$</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>3.4</td>
<td>3.8</td>
</tr>
<tr>
<td>$\mu_S$</td>
<td>1.9</td>
<td>4.0</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>7.5</td>
<td>7.8</td>
</tr>
<tr>
<td>Cost of Assets-Only allocation</td>
<td>12.0</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Note: The table reports the optimal portfolio weights corresponding to the assets-liabilities portfolio, for the global minimum variance portfolio (LHP) and for mean-variance portfolios with risk aversion $\lambda = 50$ and 20. We consider the cases ($\psi = 0.8; \pi = 1\%$) and ($\psi = 1.2; \pi = 1\%$), respectively. Log-returns are in real terms. ($\mu_A, \sigma_A$) and ($\mu_S, \sigma_S$) denote the expected return and volatility of the assets portfolio and the surplus, respectively.
Table 6: Optimal Assets-Liabilities Portfolios ($\pi = 2\%$)

<table>
<thead>
<tr>
<th>(in %)</th>
<th>$\psi = 1.0$</th>
<th>$\psi = 0.8$</th>
<th>$\psi = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LHP $\lambda = 50$</td>
<td>LHP $\lambda = 50$</td>
<td>LHP $\lambda = 50$</td>
</tr>
<tr>
<td><strong>Panel A: No weight restriction</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash</td>
<td>-111.0 -141.2</td>
<td>-67.5 -98.5</td>
<td>-132.6 -162.4</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>50.4 77.3</td>
<td>20.1 47.5</td>
<td>61.2 87.9</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>8.3 15.5</td>
<td>8.2 15.4</td>
<td>13.1 20.2</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>25.7 17.1</td>
<td>34.1 25.3</td>
<td>49.9 40.8</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>-1.1 -0.3</td>
<td>0.7 1.5</td>
<td>1.6 2.4</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>129.5 105.6</td>
<td>119.8 96.1</td>
<td>115.1 91.4</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>6.8 19.7</td>
<td>5.2 18.1</td>
<td>3.6 16.5</td>
</tr>
<tr>
<td>Commodities</td>
<td>2.3 6.9</td>
<td>0.1 4.7</td>
<td>4.1 8.6</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>-11.0 -0.5</td>
<td>-20.7 -10.0</td>
<td>-15.9 -5.4</td>
</tr>
<tr>
<td><strong>Expected return and risk:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_A$</td>
<td>2.1 4.8</td>
<td>1.3 4.1</td>
<td>2.5 5.2</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>6.7 7.3</td>
<td>5.7 6.2</td>
<td>7.4 8.1</td>
</tr>
<tr>
<td>$\mu_S$</td>
<td>1.9 4.6</td>
<td>0.9 3.6</td>
<td>2.9 5.7</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>7.2 7.6</td>
<td>4.9 5.5</td>
<td>10.4 10.7</td>
</tr>
<tr>
<td><strong>Cost of Assets-Only allocation</strong></td>
<td>20.3 9.9</td>
<td>14.1 6.9</td>
<td>25.4 12.4</td>
</tr>
<tr>
<td><strong>Cost of positivity restrictions</strong></td>
<td>8.0 5.3</td>
<td>4.1 2.7</td>
<td>11.1 7.0</td>
</tr>
<tr>
<td><strong>Panel B: Positivity restrictions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash</td>
<td>0.0 0.0</td>
<td>0.0 0.0</td>
<td>0.0 0.0</td>
</tr>
<tr>
<td>U.S. bond</td>
<td>0.0 0.0</td>
<td>0.0 0.0</td>
<td>0.0 0.0</td>
</tr>
<tr>
<td>U.S. equity</td>
<td>8.7 24.2</td>
<td>5.8 20.7</td>
<td>14.1 31.7</td>
</tr>
<tr>
<td>E.A. bond</td>
<td>25.1 17.4</td>
<td>31.0 23.6</td>
<td>50.3 41.0</td>
</tr>
<tr>
<td>E.A. equity</td>
<td>0.0 0.0</td>
<td>0.0 0.0</td>
<td>0.0 0.0</td>
</tr>
<tr>
<td>Swiss bond</td>
<td>66.3 50.2</td>
<td>63.2 46.0</td>
<td>35.7 23.8</td>
</tr>
<tr>
<td>Swiss equity</td>
<td>0.0 8.2</td>
<td>0.0 9.8</td>
<td>0.0 3.5</td>
</tr>
<tr>
<td>Commodities</td>
<td>0.0 0.0</td>
<td>0.0 0.0</td>
<td>0.0 0.0</td>
</tr>
<tr>
<td>Swiss real estate</td>
<td>0.0 0.0</td>
<td>0.0 0.0</td>
<td>0.0 0.0</td>
</tr>
<tr>
<td><strong>Expected return and risk:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_A$</td>
<td>1.2 3.2</td>
<td>1.0 3.0</td>
<td>1.7 3.3</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>3.3 3.7</td>
<td>3.3 3.7</td>
<td>3.5 4.0</td>
</tr>
<tr>
<td>$\mu_S$</td>
<td>1.0 3.0</td>
<td>0.6 2.6</td>
<td>2.2 3.8</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>8.2 8.5</td>
<td>5.7 6.1</td>
<td>11.4 11.6</td>
</tr>
<tr>
<td><strong>Cost of Assets-Only allocation</strong></td>
<td>11.4 4.6</td>
<td>9.5 4.1</td>
<td>13.4 5.5</td>
</tr>
</tbody>
</table>

Note: The table reports the optimal portfolio weights corresponding to the assets-liabilities portfolio, for the LHP and for mean-variance portfolios with risk aversion $\lambda = 50$ and 20. Log-returns are in real terms. ($\mu_A, \sigma_A$) and ($\mu_S, \sigma_S$) denote the expected return and volatility of the assets portfolio and the surplus, respectively.
Figure 1: Structure of the Global Model

- Macro Finance Model
- Demographics
- Policy Variables
- Financial Asset’s Dynamic
- Pension Fund Cash Flows and Liabilities
- Surplus Maximization
- Performance Analysis
Figure 2: Structure of the Macro-finance Model

Macroeconomic and Financial Factors

- Output gap
- Price inflation
- Employment
- Wages
- Dividend Price Ratio
- Term structure
- Cash
- Equities
- Currencies
- Long term bond

Financial Assets

Pension Fund Liabilities

- Contributions
- Benefits
Figure 3: Population and Cash Flows of the Pension Fund ($\psi = 1$)

Note: This figure displays the expected evolution of insured population for the next 40 years starting from 2014 on. Population includes employees, retired, surviving spouses, and disabled.
**Figure 4:** Population and Cash Flows of the Pension Fund ($\psi = 0.8$)

Note: This figure displays the expected evolution of insured population for the next 40 years starting from 2014 on. Population includes employees, retired, surviving spouses, and disabled.
Figure 5: Population and Cash Flows of the Pension Fund ($\psi = 1.2$)

Note: This figure displays the expected evolution of insured population for the next 40 years starting from 2014 on. Population includes employees, retired, surviving spouses, and disabled.
Figure 6: Discount Rate (in %)

Note: This figure displays the expected evolution of the discount rate for the next 40 years starting from 2014 on. The figure also displays the 20-year government bond rate.
Figure 7: Liabilities (in billion CHF)

Note: This figure displays the expected evolution of the liabilities for the next 40 years starting from 2014 on.
Figure 8: Return on Liabilities (in %)

Note: This figure displays the expected evolution of the return on liabilities for the next 40 years starting from 2014 on. The figure also displays the return on a 20-year government bond.